# Final Exam Guidelines: Material and Review Suggestions

**Date and place**: Wednesday, December 14, 3:30 – 5:30 pm, MONT 320 **Additional office hours before final exam**: Wednesday, December 14, 12:30-2:30 pm **Policies**:

- This is a one-hour exam, but all students may stay for as long as they need to finish the exam.
- UConn policy requires that any MAKE-UP for the final exam must be authorized by the Dean of Students Office.
- Make-Up Exam for students who received authorization: Thursday, December 15, 11:00 –1:00, MONT 101.

### **Material:**

• Material on which you will be tested on the Final:

Retesting: Chapter 4: 4.1, 4.2

New Material: Chapter 4: 4.3, 4.5, 4.6; Chapter 5: 5.1, 5.2, 5.3; Chapter 6: 6.1, 6.2, 6.4

• Background material you were tested on in Exam 1 and Exam 2 that you need to know in order to be able to answer questions on the final:

Chapter 1, Sections 1.3, 1.4, 1.7 (on Exam 1 Review Sheet)

Chapter 2, Sections: 2.1, 2.2, 2.3; Chapter 3, Sections: 3.1, 3.2 (on Exam 2 Review Sheet)

• Homework points = 12 points

[8 point for each new material section (6.1 + 6.2 count as one section) + 4 group-works]

- Final exam points = 88 points
- You may bring a Scientific Calculator (but not a programmable or symbolic calculator)
- You may not bring any notes or handouts

The Invertible Matrix Theorem (as in Section 2.3) will be given to you as a handout during the exam.

The exam will cover the material from the sections mentioned above that we discussed in class and studied in the homework assignments. Suggested practice exercises: THE PRACTICE PROBLEMS at the end of each section, and exercises in the same groupings as those assigned as homework problems.

## Section by section highlights of the material you should master:

#### Chapter 4

#### Section 4.1

<u>Definitions</u>: vector space, subspace of a vector space, a subspace spanned by a set of vectors

Theorems: Theorem 1 (subspace spanned by vectors, page 194)

<u>Skills</u>: determine if a set with addition and scalar multiplication is a vector space, determine if a set of vectors span R<sup>n</sup>, determine if a set is a subspace

## Section 4.2

<u>Definitions</u>: The null space of a matrix, Nul A; the column space of a matrix, Col A (both descriptions)

<u>Theorems</u>: Theorems 2, 3 (Null A and Col A are subspaces, pages 199, 201), and highlighted remark on page 202 <u>Skills</u>: Determine if a vector is in Nul A or Col A, find a non-zero vector in Nul A or Col A, find a spanning set for Nul A or Col A

#### Section 4.3

Definitions: linearly independent and dependent vectors in a vector space, basis of a vector space

<u>Theorems</u>: Theorem 4 (characterization of linearly independent vectors, page 208), Theorem 5 (The Spanning Set Theorem, page 210), Theorem 6 (basis for Col A, page 212),

Skills: determine if a set is a basis of a subspace, find a basis for Nul A, Col A, and other subspaces.

#### Section 4.5

Definitions: finite dimensional vector space, dimension of a vector space

<u>Theorems</u>: Theorem 9, 10, 11 (Number of elements in an independent set, or a basis of a space or subspace, pages 225, 226, 227), Theorem 12 (The Basis Theorem, page 227), highlighted remark on page 228

<u>Skills</u>: find the dimensions of Nul A, Col A and other subspaces, dimension of  $R^n$ , and all subspaces of  $R^n$ , geometric meaning of subspaces of  $R^n$  of dimensions 0, 1, 2, and 3.

## Section 4.6

<u>Definitions</u>: the row space of a matrix, Row A; the rank of a matrix, rank A

<u>Theorems</u>: Theorem 13 (Basis for Row A, page 231), Theorem 14 (The Rank Theorem, page 233)

<u>Skills</u>: find the dimensions and bases for Nul A, Col A, Row A, Col A<sup>T</sup> and other subspaces, determine the rank of a matrix, use the Rank Theorem

## Chapter 5

#### Section 5.1

Definitions: eigenvector, eigenvalue, eigenspace

<u>Theorems</u>: Theorem 1 (Eigenvalues of a triangular matrix, page 269), Theorem 2 (Eigenvectors of distinct eigenvalues, page 270). The remarks following Example 5, page 270 (When is 0 an eigenvalue of a matrix)

<u>Skills</u>: determine if a number (respectively, a vector) is an eigenvalue (respectively, an eigenvector) of a matrix, find the eigenvalues of a triangular matrix, find a basis for an eigenspace

#### Section 5.2

<u>Definitions</u>: the characteristic polynomial and equation of a matrix, multiplicity of an eigenvalue, similar matrices <u>Theorems</u>: Theorem (The Invertible Matrix Theorem (continued), page 275), the highlighted paragraph before Example 3, on page 276, Theorem 3 (properties of determinants, page 275)

<u>Skills</u>: find the characteristic equation of matrices, find the eigenvalues and their multiplicities of 2x2 and some 3x3 matrices using the characteristic equation, find the eigenvalues and their multiplicities of triangular matrices

## Section 5.3

Definitions: diagonalizable matrix

<u>Theorems</u>: Theorem 5 (The Diagonalization Theorem, page 282), Theorems 6 and 7 (Conditions for a matrix to be diagonalizable, pages 284, 285)

Skills: decide if a 2x2 or 3x3 matrix is diagonalizable, if A is diagonalizable find P and D such that  $A = PDP^{-1}$ 

# Chapter 6

#### Section 6.1

<u>Definitions</u>: dot (inner) product of vectors, length of a vector, orthogonal vectors, unit vector, normalization of a vector Theorems: Theorem 1 (Properties of dot product, page 331)

Skills: compute dot products, compute length of a vector, normalize a vector, decide when two vectors are orthogonal

## Section 6.2

<u>Definitions</u>: orthogonal set of vectors, orthogonal basis, orthonormal set of vectors, orthonormal basis

Theorems: Theorem 4 (Linear independence of orthogonal vectors, page 338)

Skills: check a set of vectors for orthogonality or orthonormality

## Section 6.4

Theorems: Theorem 11 (The Gram-Schmidt Process, page 355, memorize the formulas)

<u>Skills</u>: use the Gram-Schmidt process to produce an orthogonal basis from a given basis (consisting of 2 or 3 vectors), normalize the vectors of an orthogonal basis to produce an orthonormal basis.