MATH 2210Q
Fall 2010
Sarah Glaz

Final Exam Guidelines: Material and Review Suggestions

Date and place: Thursday, December 16, 10:30 – 12:30, MSB 315

Additional office hours before final exam: Monday, December 13, 5:00-6:00;
Wednesday, December 15, 11:00 – 12:00

Policies: Any MAKE-UPS for final exams must be authorized by the office of student services.
This is a one-hour exam, but all students may stay for as long as they need to finish the exam.

Material:

- **Material on which you will be tested on the Final:**
  Retesting: Chapter 4: 4.1, 4.2
  New Material: Chapter 4: 4.3, 4.5, 4.6; Chapter 5: 5.1, 5.2, 5.3; Chapter 6: 6.1, 6.2, 6.4 [you will not be tested on this section in the final exam]
- Background material you were tested on in Exam 1 and Exam 2 that you need to know in order to be able to answer questions on the final:
  Chapter 1, Sections 1.3, 1.4, 1.7 (**on Exam 1 Review Sheet**)
  Chapter 2, Sections: 2.1, 2.2, 2.3, Chapter 3, Sections: 3.1, 3.2 (**on Exam 2 Review Sheet**)
- Homework points = 11 points (1 point for each new material section + 3 group-works)
- Final exam points = 89 points
- You may bring a Scientific Calculator (but not a programmable or symbolic calculator)
- You may not bring any notes or handouts
  The Invertible Matrix Theorem (Section 2.3) will be given to you as a handout during the exam.

The exam will cover the material from the sections mentioned above that we discussed in class and studied in the homework assignments. Suggested practice exercises: THE PRACTICE PROBLEMS at the end of each section, and exercises in the same groupings as those assigned as homework problems.

Section by section highlights of the material you should master:

Chapter 4

Section 4.1

Definitions: vector space, subspace of a vector space, a subspace spanned by a set of vectors
Theorems: Theorem 1 (Spanning Set Theorem, page 221)
Skills: determine if a set with addition and scalar multiplication is a vector space, determine if a set of vectors span R^n, determine if a set is a subspace

Section 4.2

Definitions: The null space of a matrix, Nul A; the column space of a matrix, Col A (both descriptions);
Theorems: Theorems 2, 3 (Nul A, and Col A are subspaces, pages 227, 229), and highlighted remark on page 230
Skills: Determine if a vector is in Nul A or Col A, find a non-zero vector in Nul A or Col A, find a spanning set for Nul A or Col A

Section 4.3

Definitions: linearly independent and dependent vectors in a vector space, basis of a vector space
Theorems: Theorem 4 (Characterization of linearly independent vectors, page 237), Theorem 5 (The spanning set theorem, page 239), Theorem 6 (Basis for Col A, page 241),
Skills: determine if a set is a basis of a subspace, find a basis for Nul A, Col A, and other subspaces

Section 4.5

Definitions: finite dimensional vector space, infinite dimensional vector spaces, dimension of a vector space
Theorems: Theorem 9, 10, 11 (Number of elements in an independent set, or a basis of a space or subspace, pages 256, 257, 259), Theorem 12 (The basis theorem, page 259), highlighted remark on page 260
Skills: find the dimensions of Nul A, Col A and other subspaces, dimension of $R^{n,}$ and all subspaces of R^n, geometric meaning of subspaces of R^n of dimensions 0, 1, 2, and 3.

Section 4.6

Definitions: the row space of a matrix, Row A; the rank of a matrix, rank A
Theorems: Theorem 13 (Basis for Row A, page 263), Theorem 14 (The Rank Theorem, page 265), Theorem (The Invertible Matrix Theorem (continued), page 267)
Skills: find the dimensions and bases for Nul A, Col A, Row A, Col A^T and other subspaces, determine the rank of a matrix, use the Rank Theorem

Chapter 5

Section 5.1

Definitions: eigenvector, eigenvalue, eigenspace
Theorems: Theorem 1 (Eigenvalues of a triangular matrix, page 306), Theorem 2 (Eigenvectors of distinct eigenvalues, page 307). The remarks following Example 5, page 306: When is 0 an eigenvalue of a matrix
Skills: determine if a number (respectively, a vector) is an eigenvalue (respectively, an eigenvector) of a matrix, find the eigenvalues of a triangular matrix, find a basis for an eigenspace

Section 5.2

Definitions: the characteristic polynomial and equation of a matrix, multiplicity of an eigenvalue, similar matrices
Theorems: Theorem (The Invertible Matrix Theorem (continued), page 312), the highlighted paragraph before Example 3, on page 313, Theorem 3 (properties of determinants, page 313), Theorem 4 (Eigenvalues of similar matrices, page 315)
Skills: find the characteristic equation of matrices, find the eigenvalues and their multiplicities of 2x2 and some 3x3 matrices using the characteristic equation, find the eigenvalues and their multiplicities of

triangular matrices

Section 5.3

Definitions: diagonalizable matrix
Theorems: Theorem 5 (The diagonalization theorem, page 320), Theorems 6 and 7 (Conditions for a matrix to be diagonalizable, pages 323, 324)
Skills: decide if a 2x2 or 3x3 matrix is diagonalizable, if A is diagonalizable find P and D such that $A = PDP^{-1}$, show how to compute high powers of diagonalizable matrices

Chapter 6

Section 6.1

Definitions: dot product of vectors, length of a vector, distance between two vectors, orthogonal vectors, unit vector, normalization of a vector
Theorems: Theorem 1 (Properties of dot product, page 376)
Skills: compute dot products, compute length of a vector, compute the distance between two vectors, normalize a vector, decide when two vectors are orthogonal

Section 6.2

Definitions: orthogonal set of vectors, orthogonal basis, orthonormal set of vectors, orthonormal basis, (orthogonal) projection of a vector on a line or on another vector
Theorems: Theorem 4 (Linear independence of orthogonal vectors, page 384)
Skills: check a set of vectors for orthogonality or orthonormality, compute the projection of a vector onto a line or onto another vector, decompose a vector into a sum of two vectors, one in the direction of **u** and another orthogonal to **u**.

Section 6.4 [read handout, but you will not be tested on this section in the final exam]

Theorems: Theorem 11 (The Gram-Schmidt process, page 404, memorize the formulas)
Skills: use the Gram-Schmidt process to produce an orthogonal basis from a given basis (consisting of two or three vectors)