Math 195Q
Spring 2006
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Name: _____

Final Exam with Solutions

Exercise 1 Perform the indicated operations between rational expressions. Write your answers in completely simplified (reduced) form and with positive exponents only. For full credit show your work.

Exercise 1a (6 points) $\dfrac{5}{2(x-5)} - \dfrac{30}{x^2-25}$

$$\frac{5}{2(x-5)} - \frac{30}{x^2-25} = \frac{5(x^2-25) - 30\cdot 2(x-5)}{2(x-5)(x^2-25)} = \frac{5x^2-60x+175}{2(x-5)(x^2-25)}$$

Now, factor the numerator and denominator:

Denominator: $(x^2-25) = (x^2-5^2) = (x-5)(x+5)$

This makes the denominator equal to $2(x-5)(x-5)(x+5)$

Numerator: First factor out GCF $5x^2-60x+175 = 5(x^2-12x+35)$

We now find the roots of $x^2-12x+35$. $r = \dfrac{12 \pm \sqrt{144-140}}{2} = \dfrac{12 \pm 2}{2} = 5$ and 7.

We conclude that the numerator is $5(x^2-12x+35) = 5(x-7)(5-5)$

We now reduce the fraction to get the final answer

$$\frac{5x^2-60x+175}{2(x-5)(x^2-25)} = \frac{5(x-7)(x-5)}{2(x-5)(x-5)(x+5)} = \frac{5(x-7)}{2(x-5)(x+5)}$$

Exercise 1b (6 points) $\dfrac{2x^4}{5x^5y^2} + \dfrac{3y}{5xy^3}$

$$\frac{2x^4}{5x^5y^2} + \frac{3y}{5xy^3} = \frac{2x^4 5xy^3 + 3y5x^5y^2}{5x^5y^2 5xy^3} = \frac{10x^5y^3 + 15x^5y^3}{25x^5y^3xy^2} = \frac{25x^5y^3}{25x^5y^3xy^2} = \frac{1}{xy^2}$$

Exercise 2 Evaluate the following numerical expressions. Write your answers in the simplest possible way. Do not leave exponents or radical signs in your answers. For full credit show your work.

Exercise 2a (5 points) $16^{-3/2}$

$$16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64}$$

Exercise 5b (5 points) $\dfrac{5^{1/3}}{5^{4/3}}$

$$\frac{5^{\frac{1}{3}}}{5^{\frac{4}{3}}} = 5^{\frac{1}{3}-\frac{4}{3}} = 5^{-\frac{3}{3}} = 5^{-1} = \frac{1}{5}$$

Exercise 3 Simplify the following radical and exponential expressions. Assume all the variables in this exercise represent non-negative numbers. Do not leave negative exponent, or radical signs in your answers. For full credit show your work.

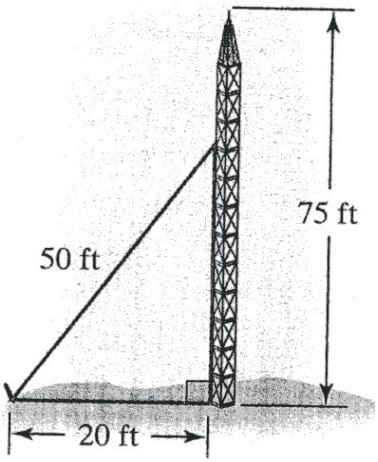
Exercise 3a (6 points) $\sqrt{49x^4y^2}$

$\sqrt{49x^4}\,y^2 = 7x^2y$ because $(7x^2y)^2 = 7^2(x^2)^2\,y^2 = 49x^4y^2$

Exercise 3b (6 points) $\dfrac{y^{-3/10}\cdot y^{3/5}}{2\cdot y^{-7/10}\cdot y}$

$$\frac{y^{-3/10}\cdot y^{3/5}}{2\cdot y^{-7/10}\cdot y} = \frac{y^{-\frac{3}{10}+\frac{3}{5}+\frac{7}{10}-1}}{2} = \frac{y^{\frac{4}{10}+\frac{3}{5}-1}}{2} = \frac{y^{\frac{2}{5}+\frac{3}{5}-1}}{2} = \frac{y^{\frac{5}{5}-1}}{2} = \frac{y^0}{2} = \frac{1}{2}$$

Exercise 4 (6 points) A 50-foot supporting wire is to be attached to a 75-foot vertically standing antenna. Because of the surrounding buildings, sidewalks, and roadways, the wire must be anchored exactly 20 feet from the base of the antenna.



How high from the base of the antenna is the wire attached?

For full credit explain how you obtained your answer and show your work.
(You may use $\sqrt{21} = 4.58$)

c = hypothenuse = *50*
b = one side = *20*
a = the other side = *?*

$a^2 + b^2 = c^2$ implies $a^2 + 20^2 = 50^2$ implies $a^2 + 400 = 2500$ implies $a^2 = 2100$ implies
$a = \sqrt{2100} = 45.8$

Exercise 5 (6 points) Use the properties of logarithms to write the following expression as one logarithm. Write your answer in the simplest possible way. Do not leave radical signs or negative exponents in your answer. For full credit show your work and explain how you obtained your answer.

$4\log_5 x + \log_5 y - 3\log_5 z - \log_5 \sqrt{x}$

$$4\log_5 x + \log_5 y - 3\log_5 z - \log_5 \sqrt{x} = \log_5 x^4 + \log_5 y - \log_5 z^3 - \log_5 x^{\frac{1}{2}}$$
$$= \log_5 \frac{x^4 y}{z^3 x^{\frac{1}{2}}} = \log_5 \frac{x^{4-\frac{1}{2}} y}{z^3} = \log_5 \frac{x^{\frac{7}{2}} y}{z^3}$$

Exercise 6 If $\log_b 5 = -1.25$ and $\log_b 2 = -0.53$, evaluate the following expressions. For full credit show your work and explain how you obtained your answer.

Exercise 6a (6 points) $\log_b \frac{5}{2}$

$$\log_b \frac{5}{2} = \log_b 5 - \log_b 2 = -1.25 + 0.53 = -0.72$$

Exercise 6b (6 points) $\log_b 2 \cdot \sqrt{5}$

$$\log_b 2 \cdot \sqrt{5} = \log_b 2 + \log_b \sqrt{5} = \log_b 2 + \log_b 5^{\frac{1}{2}}$$
$$= \log_b 2 + \frac{1}{2}\log_b 5 = -0.53 - \frac{1.25}{2} = -1.15$$

Exercise 7 Solve each of the following equations. For full credit show all your work .

Exercise 7a (6 points) $4^{2x-3} = \frac{1}{16}$

$4^{2x-3} = 4^{-2} \rightarrow 2x - 3 = -2 \rightarrow 2x = 1 \rightarrow x = \frac{1}{2}$

Exercise 7b (6 points) $log_6 x = -2$

$x = 6^{-2} \rightarrow x = \frac{1}{6^2} = \frac{1}{36}$

Exercise 7c (6 points) $\ln \frac{1}{e^5} = 3x$

$\ln \frac{1}{e^5} = \ln e^{-5} = (-5) \ln e = -5 \rightarrow -5 = 3x \rightarrow x = -\frac{5}{3}$

Exercise 7d (6 points) $log_7 4 + log_7 x = 3$

$$\log_7 4 + \log_7 x = \log_7 4x \rightarrow \log_7 4x = 3 \rightarrow 4x = 7^3 \rightarrow 4x = 343 \rightarrow x = \frac{343}{4}$$

Exercise 7e (6 points) $e^{3x-9} = 8$ (You may use $\ln 8 = 2.08$)

Take ln on both sides to get:

$$\ln e^{3x-9} = \ln 8 \rightarrow 3x - 9 = 2.08 \rightarrow 3x = 11.08 \rightarrow x = \frac{11.08}{3} = 3.69$$

Good Luck!