Math 195Q
Spring 2006
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Name: ____________________________________

Exam 3 with Solutions

Exercise 1 Simplify each of the following algebraic expressions using the exponential rules. **Do not leave negative exponents in your answer**. For full credit show your work..

Exercise 1a (6 points) $\left(\dfrac{3x^{-4}y}{x^{-3}y^{2}}\right)^{3}$

$$\left(\frac{3x^{-4}y}{x^{-3}y^{2}}\right)^{3} = \left(\frac{3x^{-4+3}\cdot y^{1-2}}{1}\right)^{3} = \left(\frac{3x^{-1}y^{-1}}{1}\right)^{3} = \left(\frac{3}{xy}\right)^{3} = \frac{3^{3}}{(xy)^{3}} = \frac{27}{x^{3}y^{3}}$$

Exercise 1b (6 points) $(x^{-1}y^{3})^{-2}(2x)$

$$(x^{-1}y^{3})^{-2}\cdot 2x = (x^{-1})^{-2}\cdot(y^{3})^{-2}\cdot 2x = x^{2}y^{-6}2x = 2x^{3}y^{-6} = \frac{2x^{3}}{y^{6}}$$

Exercise 1c (6 points) $\dfrac{x^{3m+7}\cdot x^{m-2}}{x^{m-4}}$

$$\frac{x^{3m+7}\cdot x^{m-2}}{x^{m-4}} = x^{(3m+7)+(m-2)-(m-4)} = x^{3m+7+m-2-m+4} = x^{3m+9}$$

Exercise 2 Consider the two polynomials $P(x) = 3x - 4$ and $Q(x) = 6x^2 - 5x + 1$. Calculate the following expressions, carrying out the indicated operations and simplifying your answer whenever possible. For full credit show your work.

Exercise 2a (6 points) $P(x) \cdot Q(x) =$

$$P(x) \cdot Q(x) = (3x - 4)(6x^2 - 5x + 1) = 18x^3 - 15x^2 + 3x - 24x^2 + 20x - 4 =$$
$$= 18x^3 - 39x^2 + 23x - 4$$

Exercise 2b (5 points) $P(4) + Q(-2) =$

$$P(4) = 3 \cdot 4 - 4 = 8$$
$$Q(-2) = 6(-2)^2 - 5(-2) + 1 = 35$$
$$P(4) + Q(-2) = 8 + 35 = 43$$

Exercise 2c (6 points) $Q(x) - P(x) =$

$$Q(x) - P(x) = (6x^2 - 5x + 1) - (3x - 4) = 6x^2 - 5x + 1 - 3x + 4 = 6x^2 - 8x + 5$$

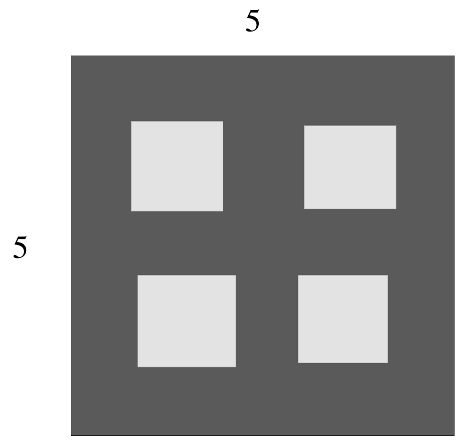
Exercise 2d (5 points) *P(-1) - Q(-1)* =

$$P(-1) = 3(-1) - 4 = -3 - 4 = -7$$
$$Q(-1) = 6(-1)^2 - 5(-1) + 1 = 6 + 5 + 1 = 12$$
$$P(-1) - Q(-1) = -7 - 12 = -19$$

Exercise 3 You have a large square of paper 5 inches by 5 inches. Out of it you cut four small squares. Each side of every one of the four small squares is of equal length, which we denote by x. To the right is a picture of the situation.

Before you answer the questions below, mark each side of the small squares by x.

Remember: The area of a rectangle is equal to its width multiplied by its length.



Exercise 3a (6 points) Write a polynomial expression in variable x for the area of EACH SMALL SQUARE that you cut out of the large square. <u>For full credit explain how you obtained your answer.</u>

The area of each square with side x is equal to x^2.

Exercise 3b (6 points) Consider the darker shaded region of the large square. This is the region left after you cut out and discard the four small squares. Express the area of this region as a function of x. Denote this area by $y = A(x)$. <u>For full credit explain how you obtained your answers</u>.

The area of the shaded region is equal to the area of the large square minus 4 times the area of the small square. $A(x) = 5^2 - 4x^2 = 25 - 4x^2$

Exercise 3c (6 points) For the area $A(x)$ you found in Exercise 3b, calculate $A(\frac{5}{2})$. You may do the calculation algebraically by using the expression found in Exercise 3b, or give your answer using a geometric reasoning. <u>In either case show your work and justify your answer</u>.

$A(5/2) = 25 - 4(5/2)^2 = 25 - 4(25/4) = 25 - 25 = 0$

Exercise 4 Factor each of the following expressions **completely**.
In each case show your work, and explain which formulas you used for the factorization.

Exercise 4a (6 points) $x^2z^2 - 64 =$

$x^2z^2 - 64 = (xz)^2 - 8^2 = (xz+8)(xz-8)$,
using the formula $a^2 - b^2 = (a+b)(a-b)$

Exercise 4b (6 points) $15x^3y - 21x^2y^2 =$

$15x^3y = 3 \cdot 5 \cdot xxxy$
$21x^2y^2 = 3 \cdot 7 \cdot xxyy$ GCF $= 3x^2y$, $15x^3y - 21x^2y^2 = 3x^2y(5x - 7y)$

Exercise 4c (6 points) $9x^2 + 30x + 25 =$

a = 9, b = 30, c = 25 $r = \frac{-30 \pm \sqrt{30^2 - 4 \cdot 9 \cdot 25}}{2 \cdot 9} = \frac{-30 \pm \sqrt{0}}{18} = \frac{-30}{18} = -\frac{5}{3}$

$9x^2 + 30x + 25 = 9(x - (-\frac{5}{3}))(x - (-\frac{5}{3})) = 9(x + \frac{5}{3})^2$

Exercise 5 Simplify completely the following rational expressions.
For full credit show your work.

Exercise 5a (6 points) $$\frac{x^2}{10x^3 - 2x^2}$$

The GCF of $10x^3$ and $2x^2$ is $2x^2$. Factoring the denominator we get $10x^3 - 2x^2 = 2x^2(5x - 1)$.

Therefore $$\frac{x^2}{10x^3 - 2x^2} = \frac{x^2}{2x^2(5x-1)} = \frac{1}{2(5x-1)}$$

Exercise 5b (6 points) $$\frac{2x^2 - 18}{x^2 - 2x - 3}$$

Factor the numerator: First factor out the GCF 2 and get. $2x^2 - 18 = 2(x^2 - 9)$
Now factor $x^2 - 9$ using the formula $a^2 - b^2 = (a + b)(a - b)$ with $a = x$ and $b = 3$
The factored numerator is: $2x^2 - 18 = 2(x + 3)(x - 3)$

Factor the denominator: first find roots of the quadratic equation. $a = 1, b = -2, c = -3$

$$r = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$$

$$r_1 = \frac{2+4}{2} = 3, \; r_2 = \frac{2-4}{2} = -1$$

The factored denominator is: $x^2 - 2x - 3 = (x - 3)(x + 1)$

$$\frac{2x^2 - 18}{x^2 - 2x - 3} = \frac{2(x+3)(x-3)}{(x-3)(x+1)} = \frac{2(x+3)}{x+1}$$

Good Luck!