## Exam 2

Exercise 1 Determine if each of the following graphs represent a function. Give a full explanation for your answer.

Exercise 1a (6 points)
Answer:
Yes, it is a function, because it passes the vertical line test (every vertical line cuts the graph at most once).


## Exercise 1b (6 points)

Answer:
Not a function, because it fails the vertical line test (there is a vertical line cutting the graph more than once).


Exercise 2 Consider the function $y=f(x)=|x|+2$.
Exercise 2a (6 points) Graph the function of Exercise 2. For full credit show your calculations of the points in the plane which lead you to the drawing of the graph.

| x | y |
| ---: | ---: |
| 0 | 2 |
| 1 | 3 |
| -1 | 3 |
| 2 | 4 |
| -2 | 4 |



Exercise 2b (6 points) What are the domain and the range of the function of Exercise 2? For full credit explain how you obtained your answers.

Domain: Xs, all numbers

Range: Ys, all numbers greater or equal to 2 .

Exercise 3 Consider the equation $3 x+7 y=13$

Exercise 3a (6 points) Complete the ordered pair (, 4), so that this pair becomes a solution of the equation $3 x+7 y=13$. For full credit show your work.

Plug in $y=4$ to obtain the $x$ coordinate: $3 x+7 \cdot 4=13 \rightarrow 3 x=-15 \rightarrow x=-5$
The answer is $(-5,4)$.
Exercise 3b (6 points) Graph the equation $3 x+7 y=13$. Show the $x-y$ table on which you based your graph.

| $x$ | $y$ |
| ---: | :--- |
|  |  |
| 0 | $13 / 2$ |
| -5 | 4 |



Exercise 3c (6 points) Express the equation of the line $3 x+7 y=13$ in slope- intercept form. Show your work.

$$
5 x+2 y=16 \rightarrow 2 y=-5 x+16 \rightarrow y=-\frac{5}{2} x+8
$$

Exercise 3d (6 points) Find the slope, and the y-intercept of the line $3 x+7 y=13$. Explain how you obtained your answer.

We read the slope as the number multiplying $\mathrm{x}:-\frac{3}{7}$, and the y coordinate of the y intercept as the lose number $\frac{13}{7}$. Thus the y intercept is: $\left(0, \frac{13}{7}\right)$.

Exercise 4 Find the equations (any form) of the following lines. For full credit show your work, and explain your answers.

Exercise $4 \mathbf{a}$ (6 points) The line passing through the points $(1,-1)$ and $(-2,5)$.
The slope is $\frac{5-(-1)}{-2-1}=-\frac{6}{3}=-2$. Using the point-slope formula we get the equation of the line to be: $y+1=-2(x-1)$

Exercise $\mathbf{4 b}$ (6 points) The line passing through ( 1,1 ), and perpendicular to the line $x+4 y=2$.

We first express the line $x+4 y=2$ in slope-intercept form. $4 y=-x+2 \rightarrow y=-\frac{1}{4} x+\frac{1}{2}$ This means the slope of this line is $-\frac{1}{4}$. Since perpendicular lines have slopes multiplying into -1 , the slope of the line we want is 4 . We want the equation of the line passing through $(1,1)$ with slope 4. Using the slope-point formula we get: $y-1=4(x-1)$

Exercise 5 (12 points) Is the line given by the equation $5 x-2 y=6$ parallel or perpendicular to the line passing through the points $(4,-1)$ and $(3,2)$ ? For full credit show your work and explain the reasons for your answer.

We first convert the equation of the line to slope-intercept form $5 x-2 y=6 \rightarrow y=\frac{5}{2} x-3$ The slope of this line is $m_{1}=\frac{5}{2}$. We now calculate the slope of the line passing through the two points $(4,-1)$ and $(3,2)$. Use the formula $m_{2}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{-1-2}{4-3}=-3$.

Check if they are parallel: Since the two slopes are not equal, the lines are not parallel

Check if they are perpendicular: Calculate $m_{1} \cdot m_{2}=(-3) \cdot \frac{5}{2} \neq-1$. Since the two slopes do not multiply to -1 , the two lines are not perpendicular either.

Exercise 6 (6 points) Find the solutions of the following system of equations. For full credit show your work.

$$
\begin{gathered}
-2 x+3 y=-2 \\
3 x-2 y=7
\end{gathered}
$$

Solution:
$-2 x+3 y=-2 \rightarrow y=\frac{2}{3} x-\frac{2}{3}$
$3 x-2 y=7 \rightarrow 3 x-2\left(\frac{2}{3} x-\frac{2}{3}\right)=7 \rightarrow 3 x-\frac{4}{3} x+\frac{4}{3}=7 \rightarrow 9 x-4 x+4=21 \rightarrow$
$5 x=17 \rightarrow x=\frac{17}{5}$
$y=\frac{2}{3} x-\frac{2}{3} \rightarrow y=\frac{2}{3} \cdot \frac{17}{5}-\frac{2}{3} \rightarrow y=\frac{8}{5}$

Solution: (17/5, 8/5)

Exercise 7 A rectangular garden is measured for a fence. It is noticed that the Length of the garden minus the Width of the garden is 4 feet. It is also noticed that the Perimeter of the garden is 20 feet.

Exercise 7a (6 points) Translate the statement of Exercise 7 into a system of two linear equations with two variables.

Denote Length by $x$ and Width by $y$. The perimeter is the addition of lengths of all sides, that is $x+x+y+y=2 x+2 y$. We now set up the system described in the Exercise:

$$
\begin{aligned}
& x-y=4 \\
& 2 x+2 y=20
\end{aligned}
$$

Exercise 7b (6 points) Solve the system of equations found in Exercise 7a, by graphing. And answer the question: What are the Length and the Width of the garden? For full credit show your work.

The intersection point of the two lines of this system is (7,3). Answer: Length 7, Width 3.

