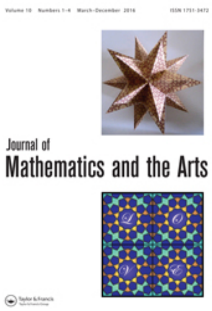

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Poems structured by integer sequences

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ABSTRACT
Poems and numbers, in particular positive integers, are intimately connected. Numbers contribute to the structure of a poem. Poetry's musicality depends not only on words, but also on quantifiable structural elements. This article explores several techniques for constructing poems using properties of sequences of positive integers. Borrowing from the name of the sequence, the poems generated by the techniques discussed in this paper can be divided into three groups: Fibonacci poems, irrational poems and prime-number poems. The paper provides an overview of each technique, including a brief mathematical background, its history, relevant literature, technical details on how it is used to structure poems and a sample of poems generated by it. We conclude with several related ideas for further exploration.

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Introduction

Poems and numbers, in particular positive integers, are intimately connected. Numbers contribute to the structure of a poem. Poetry's musicality depends not only on words, but also on quantifiable structural elements, and formal poetry relies on counting: metrical feet, rhyme words, line length, number of lines in a stanza, number of stanzas in the poem and more. It is no wonder that during the Elizabethan era, when English poetry used stricter numerical constraints than we do today, critics employed the word 'numbers' for prosody in general [21]. The use of 'numbers' as a synonym for poetic measure persisted beyond the Elizabethan era. William Wordsworth, in the ballad *The Solitary Reaper*, was still writing 'Perhaps the plaintive numbers flow....' in the nineteenth century [36]. A certain amount of numerical computation, either formal or intuitive, is involved in free verse as well [5].

In mathematics, a sequence is an ordered arrangement of objects, finite or infinite, in which repetitions are allowed. Given the close and longstanding connection between poems and numbers, it is natural to map numbers in a sequence to structural elements in a poem – letters, words, lines, stanzas, etc. And indeed, the structure of many traditional poetic forms may be considered to be associated in some way with the sequence of all the positive integers. This paper explores several techniques for constructing poems using properties of other infinite sequences of positive integers. Borrowing from the name of the sequence, the poems generated by the techniques discussed in this paper can be divided into three groups:

- Fibonacci poems: poems whose structure uses the Fibonacci sequence.
- Irrational poems: poems whose structure uses the sequence of numbers that form the decimal expansion of an irrational number, such as, for example, the number π.
- Prime-number poems: poems whose structure uses properties of the sequence of prime numbers.

The paper provides an overview of each technique, including a brief mathematical background, its history, relevant literature, technical details on how it is used to structure poems and a sample of poems generated by it. We conclude with several related ideas for further exploration.

Fibonacci poems

The Fibonacci sequence is the sequence made up of the following numbers – called Fibonacci numbers:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

Starting from the number 2 in the third position, each number in this sequence is constructed by adding the previous two Fibonacci numbers.

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The Fibonacci sequence appears in ancient Indian mathematics in connection with Sanskrit prosody. In the Sanskrit tradition of prosody, there was interest in enumerating rhythmic patterns created by the pairing of long and short syllables. Enumerating the different patterns resulted in the Fibonacci sequence [23]. The first appearance of the sequence in western mathematics literature was in 1202, in the book *Liber Abaci* [7], written by the Italian mathematician Leonardo Fibonacci. In *Liber Abaci*, the Fibonacci sequence was produced by the pairing of rabbits and represented the ideal growth of a rabbit population over time. In their book *Discovering Patterns in Mathematics and Poetry*, Marcia Birken and Anne Coon [2] describe some uses of the Fibonacci sequence in western prosody. This includes an analysis of Virgil's *Aeneid* and Dante's *The Divine Comedy*, which led to speculations of the possible use of the Fibonacci numbers or their ratios in parts of these works. Of particular interest is the mention of several experiments in contemporary poetry with the Fibonacci sequence in the count of the number of words used in each line, the number of syllables per line, or in new forms that incorporate parts of the Fibonacci sequence. Poems that involve the Fibonacci sequence as a structural element will be called Fibonacci poems.

The term Fib was coined by Gregory K. Pincus [30]. According to Pincus, a Fib is a 6-line poem of 20 syllables with each line's syllable count following the first six numbers of the Fibonacci sequence: 1, 1, 2, 3, 5 and 8. Here is one of Pincus' original Fibs:

One
Small,
Precise,
Poetic,
Spiraling mixture:
Math plus poetry yields the Fib.

Pincus' Fib, appearing in his blog in 2006 [30], attracted immediate and widespread public attention. His blog became central to this form of poetry. The Fib entry alone received 407 comments, many of them in Fib form. Pincus' blog entry seems to have revived the interest of the poetry community in the use of the Fibonacci sequence. To our delight, like the proverbial Fibonacci rabbits, Fibonacci poems are multiplying in online and print journals unchecked. Below is a Fibonacci poem by Sherman Alexie, *Requiem for a Pay Phone*, a variation of which appeared in Shampoo [1], that stops the count of syllables per line at the Fibonacci number 55, and then reverses the process to end the poem in a one-syllable line. The structure of this poem uses, in addition to the Fibonacci sequence, another mathematical structural device: it employs symmetry about an imaginary x-axis drawn through the 55-syllable line.

Requiem for a Pay Phone

A Fibonacci sequence poem

All
That
Autumn,
I walked from
The apartment (shared
With my sisters) to that pay phone
On Third Avenue, next to a sleazy gas station
And down the block from the International House of Pancakes. I was working the night
Shift at a pizza joint and you were away at college. You dated a series of inconsequential boys. Well, each boy meant little on
His own, but their cumulative effect devastated my brain and balls. I wanted you to stop kissing relative strangers, so I called at midnight as often as I could afford to. If I talked to you that late, I knew
(Or hoped) you couldn't rush into anybody's bed. But, damn, I still recall the misery of hearing the ring, ring, ring, ring, ring, ring, ring
Of your unanswered phone. These days, I'd text you to find you, but where's the delicious pain
In that? God, I miss standing in the mosquito dark
At this or that pay phone. I wish
That I could find one
And call back
All that
I
Loved.

The Fibonacci sequence is intriguing, not only because it exhibits an attractive numerical pattern, but also because it makes frequent appearance in natural settings. In particular, the Fibonacci numbers are associated with orderly arrangements in spirals exhibited by certain plants. Examples include certain sunflower seed arrangements in the flower's central disk, the spiral appearing at the base of a pinecone, and the spirals climbing upwards along the scaly rind of a pineapple. Figure 1 shows a possible tiling with squares whose side lengths are successive Fibonacci numbers and the Fibonacci spiral resulting from drawing connecting circular arcs between opposite corners of the squares.

The Fibonacci sequence is related to a visual aesthetic principle, the so-called golden ratio or divine proportion, known as Φ. If we denote the nth Fibonacci number by F_n, then

$$\Phi = \lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2} = 1.6180339\ldots$$

is the golden ratio. The golden ratio is an ancient number, which arose from a certain division of line segments, and appears in sources as old as Euclid's *Elements* [6].

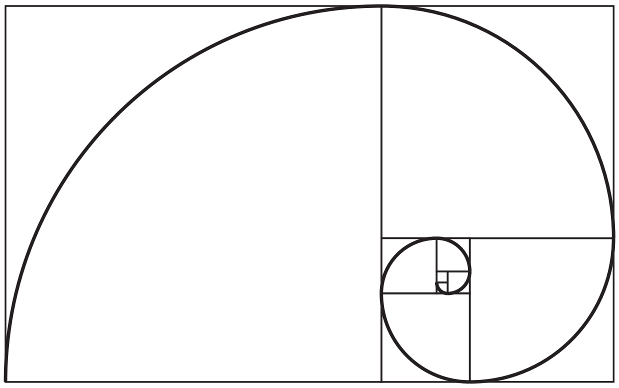

Figure 1. A Fibonacci spiral.

However, both its name — golden or divine — and its status as an ancient aesthetic principle are inventions of a number of nineteenth century authors [25]. If the ratio of the long side of a rectangle to its short side is equal to the golden ratio, the rectangle is called a golden rectangle. Among the myths and misconceptions about the golden ratio are the beliefs that golden rectangles are 'most pleasing' in form of all rectangles, and that close approximations of them appear in many notable past visual works of art and architecture [25]. Many contemporary artists create work around the golden ratio, often because they are captivated by the mythology surrounding it. It is an interesting design choice, even without attributing to it mystical significance or ancient tradition.

When we start with a golden rectangle and remove from it a square whose sides are equal to the short side of the rectangle, we are left with a new, smaller, golden rectangle. This process may be continued for as long as we wish. If the squares are marked rather than removed, the process yields a tiling of the large golden rectangle by smaller golden rectangles. It is then possible to fit a logarithmic spiral to the same square corners visited by the Fibonacci spiral. This particular logarithmic spiral is also known as the golden spiral. In each rectangle appearing in Figure 1, the ratio of the long side to the short side is equal to $\frac{F_{n+1}}{F_n}$ for some n. Since $\frac{F_{n+1}}{F_n}$ approximates Φ, such a rectangle is an approximation of a golden rectangle, and the resulting Fibonacci spiral is an approximation of the golden spiral. A tiling of similar nature inspired the Fibonacci poem, *Spiral*, by Emily Galvin (Figure 2) [8]. To read the poem, follow the author's note: 'The key to reading it straight through is finding the curve of Fibonacci and the golden ratio.'

A modern classic is the Fibonacci poem *Tjanting*, by Ron Silliman [31]. *Tjanting*, originally published in 1981 and still in print today, is a book length prose poem where the Fibonacci sequence determines the number of sentences in each paragraph and controls the feel of the work as a whole. The poem spans 200 pages — a total of 19 paragraphs — and ends at the Fibonacci number 4181. This complex work is considered to be a major exemplar of the school of writing called Language Writing, and as such it is discussed at length in many Internet sources. It is interesting to note that the 19th paragraph's first sentence is: 'What makes this the last paragraph?' — a question with no answer that prompts us to contemplate our inborn need to strive for the infinite in its many forms.

Emily Galvin's book, *Do the Math* [8], includes more Fibonacci poems. For additional Fibonacci poems, see, for example, *Strange Attractors: Poems of Love and Mathematics* [17], which includes Judith Baumel's *Fibonacci* (in which stanza lengths follow the Fibonacci sequence), an excerpt from the book length poem *alphabet*, by Danish poet Inger Christensen (where the lengths of sections follow the pattern of the Fibonacci sequence), and Kathryn DeZur's *Fibonacci Numbers* (a poem that uses the Fibonacci sequence as metaphor rather than as structural element).

Irrational poems

Every irrational number has an infinite and non-repeating decimal expansion. For example, the number π can be expressed as

$$\pi = 3.14159265358979\ldots.$$

The number π, the ratio of the circumference of a circle by its diameter, is the most celebrated irrational number. It has captured the imagination of mathematicians, artists and the general public alike. 14 of March (denoted in the US by 3/14) became the National π Day, and the quests for a larger and larger number of digits after the decimal point and for possible hidden patterns in π's decimal expansion continue to this day [35]. π has also been the subject of several poems, including *Pi* by Nobel Prize Laureate Wislawa Szymborska [33].

The literary movement known as Oulipo — Ouvroir de Litterature Potentielle (Workshop of Potential Literature) — was founded in France by Raymond Queneau in 1960. Its members invented constraints that generate literature, many of them mathematical. Jacques Bens (1931–2001), a member of the Oulipo group, invented a new use for π in poetry — a use that involves its decimal expansion in the construction of a poetic form he called 'irrational sonnet.'

A sonnet is a 14-line poem and in its classical form it is either Petrarchan or Shakespearean. The Petrarchan sonnet has two stanzas: an octave (8 lines) and a sestet (6 lines). The Shakespearean sonnet has four stanzas: three quatrains (4 lines each) and a couplet (2 lines). Each form has a fixed metre and rhyme scheme. Adding the first five digits in the decimal expansion of π, we obtain $3 + 1 + 4 + 1 + 5 = 14$, precisely the number of

SPIRAL

I	F	E	E	L	L	I	K	E	T	H	E	S	G	L	O	V	E	B	O	X
M	A	R	T	E	S	T	K	I	D	O	N	T	.	S	U	R	E	,	Y	O
H	E	S	H	O	R	T	B	U	S	.	I	M	U	R	E	L	E	C	T	R
P	O	S	S	I	B	L	E	R	E	L	A	T	O	N	I	C	L	A	D	Y
I	O	N	S	H	I	P	S	.	T	H	E	W	J	U	M	P	S	T	H	E
H	E	E	L	I	S	S	P	I	N	N	I	N	G	U	N	F	R	O	M	T
G	B	U	T	T	H	E	H	A	M	S	T	E	I	M	E	T	O	T	I	M
R	L	O	N	G	S	I	N	C	E	D	I	E	E	,	&	S	T	I	L	L
D	.	T	H	I	S	T	I	M	E	T	H	E	I	S	T	I	N	S	T	R
Y	D	E	C	I	D	E	D	T	O	M	A	K	A	N	I	U	M	E	N	T
E	T	H	E	D	E	A	F	K	I	D	P	L	I	N	S	—	I	N	T	E
A	Y	M	U	S	I	C	A	L	C	H	A	I	I	D	I	L	L	E	C	T
R	S	.	N	I	G	H	T	F	A	L	L	S	O	U	S	I	N	T	H	E
I	N	T	H	E	D	I	T	C	H	.	I	S	T	H	I	S	E	X	I	S
T	E	N	T	I	A	L	O	R	D	E	V	O	U	T	?	T	H	E	R	E
W	A	S	A	T	I	M	E	W	H	E	N	I	T	H	O	U	G	H	T	P
A	N	I	C	W	A	S	D	I	V	I	N	E	.	T	R	O	U	B	L	E
I	S	,	I	C	A	N	'	T	R	E	C	O	G	N	I	Z	E	F	I	G
M	E	N	T	S	O	F	Y	O	U	R	I	M	A	G	I	N	A	T	I	O
N	.	A	B	J	E	C	T	S	U	B	J	E	C	T	I	V	I	T	Y	,
A	S	T	H	E	M	A	N	M	I	G	H	T	S	A	Y	.	L	O	N	G
F	E	L	L	O	W	S	A	I	D	,	"	T	H	E	L	E	A	V	E	S
O	F	M	E	M	O	R	Y	S	E	E	M	E	D	T	O	M	A	K	E	A
M	O	U	R	N	F	U	L	R	U	S	T	L	I	N	G	I	N	T	H	E
D	A	R	K	.	"	L	E	A	V	E	S	,	A	S	H	E	S	,	A	R
O	S	E	B	Y	A	N	Y	O	T	H	E	R	N	A	M	E	—	T	H	E
P	O	I	N	T	I	S	,	H	E	C	A	N	'	T	R	E	M	E	M	B
E	R	W	H	E	R	E	H	E	S	P	E	N	T	T	H	E	N	I	G	H
T	.	T	H	I	S	I	S	W	H	A	T	H	A	P	P	E	N	S	W	H
E	N	Y	O	U	'	R	E	M	A	K	I	N	G	O	T	H	E	R	P	L
A	N	S	.	T	H	E	S	E	A	R	E	T	H	E	A	S	P	I	R	A
T	I	O	N	S	O	F	T	H	E	H	O	L	Y	G	H	O	S	T	.	A
N	Y	O	N	E	C	A	N	F	I	N	D	O	U	T	W	H	E	N	W	E
W	E	R	E	E	A	T	E	N	B	Y	T	H	E	S	A	I	N	T	S	.

Figure 2. *Spiral* by Emily Galvin [8].

lines in a sonnet. Jacques Bens' irrational sonnet has 5 stanzas of lengths following the first five numbers in the decimal expansion of π: 3, 1, 4, 1 and 5. He also required the irrational sonnet to follow a fixed metre and rhyme scheme. The details of these requirements may be found in the online journal *Drunken Boat* (Oulipo section, 2006) [3]. Below is one of Jacques Bens' irrational sonnets, *The Presbytery Has Lost None of Its Charm*, translated by Laurence Petit and Ravi Shankar [3,18].

The Presbytery Has Lost None of Its Charm

The presbytery has lost none of its charm
Nor how a garden's radiance can disarm,
Restoring hand to dog, and bridle to stallion:

But this explanation fails this mystery.

A plague on insight that cracks your talons,
The analysis that dispels your sense of alarm,
Wearing a preposterous cop's cap for a perm,
Pointing out here the just and there the felon.

No explanation can redeem a mystery

I prefer the faded charms of the presbytery
And the sham radiance of a famous garden;
I prefer (it's in my nature) the shuddery
Of fear obliterated by this tiny thief's particularity
to blatancy and fame, like some lamp of Aladdin.

Variations on Jacques Bens' idea, that is, poems whose stanza lengths follow the sequence of the decimal expansion of π, may be called irrational poems. A free verse poem following π's expansion for two more digits than an irrational sonnet is *3.141592…* by Peter Meinke [17, 27].

3.141592 …

… and Cetus chained Andromeda until he was
slain by Perseus, who then married her …

In school I was attracted
to irrational numbers
stretched out like variable stars

across an expanding void …

They're just trouble Miss MacDougall
said trying to lure me back
to *a* over *b* something above zero
we could get hold of

solid as carrots and good for our eyes

but I was a born radical
and hunted numbers salty
as Cetus the sea monster
who waylaid maidens in his
starry lair I wanted to run
in the wrong circles
like the Princess Andromeda
chained unseen in the skies
of my childhood but whose story
and happy ending I believed
with all my absurd heart because
I longed for something like
the square root of love
which I thought would be

before I passed my prime
permanent and easy as pi …

Manfred Stern's poem, *The First Digits of* π [9], is another example of an irrational poem.

The decimal expansion of π gave rise to another, much stricter, constraint for creating poetry and prose: Pilish — a style of writing in which the lengths of consecutive words match the digits in the decimal expansion of π. This technique, originally used to create mnemonics for the first few digits in the decimal expansion of π, became a writing style in the hands of one of its top practitioners: mathematician, software engineer and author of works of constrained writing, Mike Keith. Mike Keith's website [22] contains a veritable treasure trove of ideas and literary pieces created in Pilish, including several poems and a link to his book, *Not a Wake: A Dream Embodying π's Digits Fully for 10,000 Decimals.*

The digits appearing in the decimal expansion of π are not the only numbers associated with π that gave rise to poems. Tatiana Bonch-Osmolovskaya's paper [4] contains a number of poems and poem fragments whose structure or content, or both, follow various other π-related numbers. Particularly attractive is her poem, *By the Father's Order Sent to the School of Mathematics I Perceive the Beginning of Transcendence*, in which each of the 14 haiku-like stanzas represents an aspect of π. This poem appears in full in the *Bridges 2013 Poetry Anthology* [12].

Poems based on the decimal expansions of other irrational numbers have appeared in the literature with much less frequency. JoAnne Growney [19] offers the lines below as a start for a poem which creates a portrait. It is based on the decimal expansion of $\sqrt{2} = 1.41421\ldots$

1 is the number of me, when I am entire.
.
4 is the number of my children,
1 is the number of books I can read at a time,
4 is the corners of a rectangle,
2 is the chairs on my porch,
1 is how many calluses I have from holding a pencil too tight.
…

$\sqrt{2}$ has its appeal, especially for mathematicians, for the scandal it created around 500 BCE, when the Pythagorean, Hippasus, discovered that it is an irrational number. This discovery shattered the belief of the Pythagoreans that whole numbers and their ratios form the underlying order of the universe, and also invalidated some of their geometric proofs. For his sin, during a sea voyage, Hippasus was thrown overboard to a watery grave [29]. Such a dramatic start should encourage more use in poetry.

Two other famous irrational numbers are the Euler number $e = 2.71828182845\ldots$ (the base of the natural log function, ln), and the golden ratio Φ (discussed in the previous section). The history of e appears in my poem, *The Enigmatic Number e* [16], and Φ is celebrated in Rafael Alberti's sonnet, *To Divine Proportion* [17]. To the best of my knowledge, no poem has yet been created using e's or Φ's decimal expansions as a structural element. There is more fun to be had with this poetic form.

Prime-number poems

Prime numbers are positive integers larger than 1 which are divisible only by 1 and themselves. The list of the first 25 primes — all the prime numbers smaller than 100 — is given below:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97$$

Prime numbers have important characteristics that impact many areas of pure mathematics and also have practical applications, for example, to coding theory. They also come surrounded by an aura of mystery, since in spite of their deceptive simplicity, there are still many open problems related to prime numbers that defy proof or refutation. In particular, there is no algorithm that easily yields prime numbers, and neither is the question of the amount of prime numbers in any given interval of numbers settled. Moreover, the primes thin out as we search among larger and larger numbers, but their number is infinite. Euclid's proof of the infinitude of prime numbers, circa 300 BCE, is considered to be one of the most elegant proofs in mathematics. Euclid recorded his proof in the *Elements* [6].

Aspects of the links between prime numbers, culture and poetry have been explored by a number of authors. For a small sample, see references [10,11,13,14,18,34]. In his essay, *Poetry and Primes* [34], Daniel Tammet points out the use of the prime number sequence in the classical forms of Japanese poetry, the haiku and the tanka. The haiku is a 3-line poem, whose lines contain 5, 7 and 5 syllables — a total of 17 syllables. The tanka is a 5-line poem, whose lines contain 5, 7, 5, 7 and 7 syllables — a total of 31 syllables. For both poetic forms, the number of syllables per line and the total number of syllables per poem are prime numbers. There are many excellent Japanese classical haikus and tankas in English translation, but the form is still in use and it has been adopted by poets who write in English. Below is my poem, *Reflection about the t-axis*, which is constructed from two haikus:

Reflection about the *t*-axis

A bag full of words.	A bag full of words.
Tomorrow – my favorite.	*Yesterday* – my favorite,
May I have it, please.	May I have it, please.

Of recent vintage is an innovative use of the prime number sequence by Emily Galvin. In her poem, *In the Nick* [8], Emily Galvin uses the prime number sequence to mark absence rather than presence. The author added the following note for the reader of her poem:

> The unit of measure is space — nicks — between words. The space before the line, the space at the end of the line, and the space between words are counted. The number of spaces in each stanza is prime, and the poem carries out the sequence of prime numbers: 2, 3, 5, 7, 11, 13, etc.

In the Nick

There.

Over there.

Do you see it?

It's coming over this way now.

A big anvil cloud of heaven and of ash and

God only knows. A thundercloud of wonderment, some steel in the air.

Don't know where you've been but I bet you've never been anywhere where the horizon's real.

Little school of clouds like fishes in the sky: he said that they were the harbingers of hurricanes

In the Gulf. The water deep and blue inside the seashells of my ears. I never learned to swim despite everyone's best

Efforts, not even now. Look at the front. It's breathing on the mountaintops. He never said it in so many words. Truthfully, I was a little girl, and

How should I remember anything but fingertips, monsters made of cloud? That old anvil cloud is counting time. Gray and white flashes inside now and then—here it comes now, ready

Or not. This time—who knows?—it could all come down in a shower of rain and fire. This burning wheel could be the real thing. Did you see the flash? Count between the lightning and the sound—

Feel the space between you and the air. Hold your breath and count down with the storm. Did you hear that? Nearer. Up on the ridge, there's a tree that lightning struck and cleaved it clean in half. My mother

Swore that once the lightening came inside, running down the stovepipe to the kitchen where she stood. She says that sparks jumped off the stove and she could feel the air start humming loud. We should go inside, storm's about to break.

Open the windows and listen—nothing but a few taps on tin and then it's only sound and fire. Maybe it will come. Wait. Looks like it's raining on Cheyenne. Maybe we'll get some of that. Did you hear that? Listen, maybe we'll get some of that.

The group of poems that utilize the prime number sequence as structural elements may be collectively called prime-number poems. The last technique we will discuss for generating such poems involves the Fundamental Theorem of Arithmetic – another theorem about prime numbers proved by Euclid in the *Elements* [6]. The Fundamental Theorem of Arithmetic states that every positive integer larger than 1 is either a prime number or can be expressed in a unique way as a product of prime numbers. For example, the number 180 can be expressed as $180 = 2 \times 2 \times 3 \times 3 \times 5$. It is customary to group together copies of the same prime using exponents, that is, $2 \times 2 = 2^2$, and to arrange the primes in increasing order from left to right. Thus, $180 = 2^2 \times 3^2 \times 5$. To construct a poem using this theorem, one follows the steps:

- Choose the length of the poem and assign numbers to the poem's lines consecutively from bottom to top, starting at 2.
- Choose a word that stands for multiplication and a word that stands for exponentiation.
- Write the lines marked by prime numbers.
- Construct the rest of the lines in the same way that the prime numbers build the positive integers. The finished poem does not need to display the numbers associated with each line.

The first poem written with this structure – in the 1970s – was Carl Andre's poem, *On the Sadness* [17]. Another poem using this technique is JoAnne Growney's, *We are the Final Ones* [18]. My poem, *13 January 2009*, which appeared first in Recursive Angel [15], was also made using this approach [11–13].

13 January 2009

$12 = 2^2 \times 3$	Anuk is dying for Anuk is dying in the white of winter
11	The coldest month
$10 = 2 \times 5$	Anuk is dying in the falling snow
$9 = 3^2$	The white of winter for Anuk is dying
$8 = 2^3$	Anuk is dying for the white of winter
7	The drift of time
$6 = 2 \times 3$	Anuk is dying in the white of winter
5	The falling snow
$4 = 2^2$	Anuk is dying for Anuk is dying
3	The white of winter
2	Anuk is dying
1	.

In this poem, the word *in* stands for multiplication, and the word *for* stands for exponentiation. The poem is generated from the prime numbered lines: 2, 3, 5, 7 and 11, which are written first. To construct the lines that were not assigned prime numbers, first factor each nonprime line's number into a product of powers of distinct primes. For example: $12 = 2^2 \times 3$. The primes appearing in the number 12, arranged in increasing order, are 2 and 3. Line 2 is: *Anuk is dying*, and line 3 is: *The white of winter*. To construct line 12, replace the number 2 with line 2, the number 3 with line 3, multiplication with *in* and exponentiation with *for*. This makes line 12: *Anuk is dying for Anuk is dying in the white of winter.* The same procedure is used to generate the other lines of the poem. When the poem is read aloud, the echo created by the repetition of prime-numbered lines evokes an elegiac mood. Note that reading the poem from bottom to top is also a poem. This characteristic is common to all the poems constructed with this technique.

Additional poems about primes and their properties may be found in *Ode to Prime Numbers* [11]. The golden ratio, the Fibonacci numbers, $\sqrt{2}$, prime numbers and other mathematical concepts play hide and seek in the enchanting mathematical hymns of Marco Lucchesi [24].

Concluding remarks

Preference and space considerations led to the exclusion from this paper of poems that do not involve words. There are many poems that involve numbers and mathematical symbols exclusively, which utilize integer sequences for their arrangements. As an example of this genre, we conclude this paper with a representative poem, *Symphony No. 3*, by Royal Heath (Figure 3), a numerical poem that uses as a structural element the sequence of all positive integers. The poem is an excerpt from the *Heath's Number Symphonies*, appearing in Jerome B. Meyer's book, *More Fun for the Family* [28]. According to Dick Higgins [20], the work cannot be dated, but it appears to be from the nineteenth century.

An interesting observation made by a reader of this paper is that the poems included here call attention to their structure in direct or indirect ways. This practice follows the Oulipo tradition whose member, Jacques Roubaud, wrote that 'a text written under some restriction speaks of this restriction' [4]. As the name of the Oulipo movement implies, the potentiality of a structural constraint to generate literature is as important as

SYMPHONY NO. 3

$$1+2=3$$
$$4+5+6=7+8$$
$$9+10+11+12=13+14+15$$
$$16+17+18+19+20=21+22+23+24$$
$$25+26+27+28+29+30=31+32+33+34+35$$
$$36+37+38+39+40+41+42=43+44+45+46+47+48$$
$$49+50+51+52+53+54+55+56=57+58+59+60+61+62+63$$
$$64+65+66+67+68+69+70+71+72=73+74+75+76+77+78+79+80$$
$$81+82+83+84+85+86+87+88+89+90=91+92+93+94+95+96+97+98+99$$

In the above number symphony note the following facts:

1. The number of terms to the left of the equal sign is always one greater than the number of terms to the right of the equal sign and the product of these will always give you the middle number. Thus, in the first case ($1 + 2 = 3$) we had two terms to the left and one on the right and the product of these gives the middle number, 2. $4 + 5 + 6$ equals $7 + 8$ —three terms times two terms equals 6, which is the middle number. In the next row we have four terms times three terms which is 12, or the middle number.

2. Note that all the first numbers are perfect squares—that is, 1, 4, 9, 16, 25, etc. Note also that the difference between any number and the number directly above it is always the same for each row and progresses by 2. For example: to go down the middle numbers, 2, 6, 12, 20, 30, etc., we have the difference 4, 6, 8, 10, 12, etc. This is true for any of the other numbers, as you can easily verify.

Not only is this a beautiful number symphony, created by Royal Heath, but it includes every number in the entire number system linked together in a gorgeous pattern of truth.

Figure 3. *Symphony No. 3* by Royal Heath.
Source: J.B. Meyer, *More Fun for the Family*, 1938 [28, p. 128].

any single work of art created using it. And therefore, a new or non-standard structural constraint needs to be shared with the reader along with the poem structured by it. For many poets who enjoy reading and creating works with mathematical constraints, comments on the structure of a poem, especially one that cannot be easily discerned from reading the work itself, are gifts. In the preface to his volume of poetry, *A Mid-Season Sky* [26], Oulipo member Harry Mathews wrote:

> To anyone who has not experienced it, describing the obsessive fascination that can derive from a freshly discovered structure or procedure . . . . is rather like communicating the nature of a wine to someone with no sense of smell. I can only assert that its succulence is real.

Modern prosody freed from the tight constraints of the past, regarding both structure and content, celebrates free verse and freedom of expression. This freedom included forging bridges to other disciplines, notably mathematics, and using its concepts to create new types of constraints that result in new poetic forms and new sounds. The structural constraints discussed in this article fall into this category. And the subject is far from being exhausted. I invite the poets who read this article to consider exploring the potential of other integer sequences. *The On-line Encyclopedia of Integer Sequences* [32], which contains to date about 200,000 entries, is a good place to start.

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