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COHERENCE, REGULARITY AND HOMOLOGICAL DIMENSIONS OF COMMUTATIVE FIXED RINGS

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ABSTRACT

Let R be a commutative ring, let G be a group of automorphisms of R and denote by R^G the fixed subring of R. $R^G = \{x \in R \mid g(x) = x \text{ for all } g \in G\}$. This paper is concerned with the exploration of conditions under which finiteness properties such as Noetherianess and coherence; and related homological properties such as "small", weak and global dimensions (like Von Neumann regularity, semihereditarity and hereditarity), and coherent regularity descend and, to a lesser degree ascend, between R and R^G.
The paper presents a survey of the literature, some of the results proved in Glaz[19], and several new results and open problems in the area.

1. Introduction

In a survey paper entitled "Effective invariant theory", Vinberg[36] points out the conditions under which research in invariant theory becomes most effective "In the 19th century it had already become clear that an explicit solution of the fundamental problems of invariant theory was possible only in a comparatively few cases. Therefore, one of the directions in modern invariant theory is to single out, in some sense, "all" such cases and to actually solve problems of invariant theory in these cases." The present article is a combination of survey of literature, the authors' own published results and some new results and open problems, on topics related to some finiteness and homological properties of fixed rings (rings of invariants), restricted to a class of rings and groups that make the theory effective.

We denote a commutative ring by R, G will denote a group of automorphisms of R and R^G the fixed subring of R. The restriction on the extension $R^G \subset R$ that makes the investigations presented in this paper effectively solvable, is the existence of a module retraction map from R to R^G. This map appears under various names in the literature such as averaging map (Bergman[4]), or Raynolds operator (Hochster & Roberts[23], and Stanley[34]). Under this restriction, the paper is concerned with descent and ascent, between R and R^G, of a variety of finiteness and related homological properties.

Section 2 explores descent and ascent of Noetherianess and coherence, as well as surveys the early, motivating, exploration of descent of finite generation as an algebra over a field. Section 3 explores descent of regularity, primarily in case the ring is coherent. Related to coherent regularity, this section also explores the behavior of the weak dimensions of the rings involved, in particular, descent of Von Neumann regularity, semihereditarity (and hereditarity). The section concludes with a survey of literature on descent of Cohen Macaulayness and related properties in the Noetherian setting, and a conjecture in this direction for the coherent case.

2. Noetherianess and coherence

The question of descent of finiteness properties from R to R^G has its roots in the classical investigation carried out in order to solve Hilbert's fourteenth problem.

HILBERT'S FOURTEENTH PROBLEM : *Let K be a field and let G be a group of automorphisms of the polynomial ring in n variables over K,* $K[x_1,\dots,x_n]$, *fixing K. Is* $K[x_1,\dots,x_n]^G$ *a finitely generated K algebra?*

Hilbert himself posed a slightly different version of this problem in 1900 at the Paris Congress. E. Noether, Zariski, and Nagata provided a pretty complete answer to the problem.

In two papers, E. Noether[3], in 1916 and 1926, treating separately the cases where K has characteristic 0 or $p > 0$, answered the question affirmatively for a finite group G. This result holds true for a general commutative ring K.

In 1953, O. Zariski[30] answered the question affirmatively if $\mathrm{tr.deg}_K L \le 2$, where L is the field of quotients of $K[x_1,\dots,x_n]^G$.

In 1959 Nagata[30] provided a family of counterexamples for the various formulations of Hilbert's fourteenth problem. For the particular formulation given above he provided a counterexample for $\mathrm{tr.deg}_K L = 4$.

Some of the gaps left between these results, as well as other significant generalizations, were solved in the intervening years. One can obtain further information from, for example, Nagata[30], Vinberg[36], Popov[32].

More modern investigations considering descent or ascent of finiteness revolved around the Noetherian property. Here one can find descent results in Bergman[4,5] (1971), Chuang & Lee[8] (1977), McConnell & Robson[27] (1977), Montgomery[28] (1980), Fisher & Osterburg[11] (1980); and ascent results in Brewer & Rutter[7] (1977), Farkas & Snider[10] (1977), Montgomery[28] (1980). We will mention some of these results later in this section.

Let A ⊆ B be two rings, A is called a <u>module retra</u> <u>of B</u> if there is an A module homomorphism φ: B → satisfying $\varphi(a) = a$ for every element a of A. If such a m exists we will call it a <u>module retraction map</u>.

Note that if A is a module retract of B, then t module retraction map φ splits the identity map of A, th B contains A as an A module direct summand, in particular is a pure submodule of B.

Two known cases where R^G is a module retract of R a pointed out in Bergman[4] :

1. G is a finite group and o(G), the order of G, a unit in R. In this case for every x in R $\varphi(x) = (1/o(G))\Sigma\, g(x)$, where the sum is taken ov all $g \in G$.
2. G is a locally finite group (that is for every $x \in R$ the orbit of x, Gx has finite cardinalit $n(x) < \infty$), and n(x) is a unit in R for every x in R. In this case for every x in R $\varphi(x) = (1/n(x))\Sigma\, y$, where the sum is taken ove all y in Gx.

The map φ is called in Bergman[4] an averaging map, a in Hochster & Roberts[23] or Stanley[34] , a Raynolds operator. I seems that the existence of such a map is a necessa condition for any significant descent of properties from to R^G to occur.

THEOREM 1 (Bergman[4]). *Let R be a Noetherian ring. I R^G is a module retract of R then R^G is a Noetherian ring.*

That this result cannot be improved much one can se by considering the example of a finite group G and Noetherian ring R with no o(G)-torsion which fails to descen Noetherianess to R^G, given by Nagarajan[30] and sharpened b Chuang & Lee[8] (Example 1), and the example of an infinite cyclic group G and a P.I.D. R which fails to descend Noetherianess to R^G given by Bergman[4] (Example 3).

EXAMPLE 1 (Nagarajan[29], Chuang & Lee[8]).
Let $F = Z_2(a_i, b_i,\ i \geq 1)$, where Z_2 is the prime field of characteristic 2, and a_i, b_i , $i \geq 1$ are infinitely many indeterminates. Set $p_i = a_i x + b_i y$ and $R = F[[x,y]]$. Define an automorphism of R, g, by: $g(x) = x$, $g(y) = y$, $g(a_i) = a_i + yp_{i+1}$, $g(b_i) = b_i + xp_{i+1}$, $i \geq 1$. Let $G = \langle g \rangle$, then $o(G) = 2$, but 2, of course, is not a unit in R. Nagarajan had shown that R^G is not a Noetherian ring.
Chuang & Lee sharpened the example as follows:
Let $A = Z[a_i, b_i,\ i \geq 1]$, where a_i, b_i are indeterminates over Z, and let $F = A_{2A}$. A is a U.F.D. and 2A is a prime ideal of height 1, hence F is a discrete valuation ring. Set $R = F[[x,y]]$, which is a Noetherian domain since F is a P.I.D. An automorphism of R is given by $g(x) = -x$, $g(y) = y$, $g(a_i) = -a_i + yp_{i+1}$, $g(b_i) = b_i + xp_{i+1}$. Then $G = \langle g \rangle$ and $o(G) = 2$, R has no 2-torsion. As R/2R is isomorphic to the ring in Nagarajan's example, R^G is not a Noetherian ring.

In case R is a coherent ring we obtain the following descent results:

THEOREM 2 (Glaz[19]). *Let R be a coherent ring and let G be a group of automorphisms of R, then R^G is a coherent ring in the following cases:*

1. *R^G is a module retract of R, G is a locally finite group and R is a flat R^G module.*
2. *R^G is a module retract of R and R is a finitely generated R^G module.*

This is the best result one is able to obtain for descent of coherence except of some odd cases (Glaz[19]). A

series of examples were constructed in Glaz[19] to show t
the conditions of Theorem 2 cannot be relaxed.

EXAMPLE 2. Finite Groups

Let K be a field of characteristic not equal to 2, $R = K[x, y, z_i, i \geq 1]$, where $x, y, z_i, i \geq 1$ indeterminates. Construct an automorphism of R fixing K, by setting $g(x) = -x$, $g(y) = -y$, $g(z_i) = -z_i$. Let G = < then $o(G) = 2$ is a unit in R, so that R^G is a module retr of R. $R^G = K[x^2, y^2, z_i^2, xy, xz_i, yz_i]$. R is a coherent ri but the ideal $(xy : x^2) = (y^2, xy, yz_i)$ is not finit generated in R^G, and so R^G is not a coherent ring.

Nagarajan's example provides a weaker example of same phenomena in rings of characteristic equal to 2, si in this case 2 is not a unit in R. R^G of Nagarajan's exam is also not a coherent ring. The proof of this fact is to Heinzer (see Glaz[19]).

EXAMPLE 3. Infinite Groups.

The ideas behind the construction of this family examples go back to Gilmer[12], and Bergman[4].

Let A be an integrally closed domain with field quotients K. For a valuation v of K, let K_v be the valuati ring of v. Since A is integrally closed $A = \cap K_v$ over a valuations v such that $A \subseteq K_v$. Let t be an indeterminate ov A, then v extends to a valuation of A[t] via $v(a_n t^n + \ldots + a_0) = \min v(a_i)$, and hence v extends to valuation of K(t). Let $R = \cap K(t)_v$, where $K(t)_v$ is t valuation ring of the extension of v to K(t). R is a Bezo domain, (Bergman[4]), and hence, a coherent ring. Let B be subring of A. For an element b in B, define a map T_b: $A[t] \rightarrow A[t]$ by $T_b(t) = t + b$. T_b can be extended to automorphism of R. Set $G = (T_b)$, where b runs over all t elements of B. If B is infinite, then $K(t)^G = K$, and thus $R^G = A$.

Depending on our choice for A and B we can obtain variety of examples. Let k be a field and let x, y be ıdeterminates over k. Set $A = k[y, x^i y]$. A is not a coherent ing as the ideal $xyA \cap x^2yA = (x^2y^2, x^3y^2, x^4y^2, \ldots)$ is not initely generated. A is integrally closed.

If k is an infinite field of characteristic $p > 0$, and = k then every element of G has order p, and, thus we btain an example where R is coherent of characteristic > 0, G is an infinite group whose elements are of finite rder and $R^G = A$ is not coherent.

If we pick B = k and the characteristic of k is zero hen no element of G has finite order.

We can modify this construction to obtain even sharper ·xamples. Let A be a non coherent Krull domain of :haracteristic zero. Such a ring is constructed in :akin & Heinzer[9]. We follow the above construction utilizing ı$[s, t, s^{-1}, t^{-1}]$ for indeterminates s and t, instead of A[t]. [n this case R is actually a P.I.D. and G may be chosen to ɔe infinite cyclic. We therefore obtain an example of a Noetherian (P.I.D.) ring R, an infinite cyclic group G and $R^G = A$ is not a coherent ring.

Regarding ascent of finiteness conditions Brewer & Rutter[7], and Farkas & Snider[10] proved independently the following result:

THEOREM 3 (Brewer & Rutter[7], Farkas & Snider[10]). *Let R be a reduced ring and let G be a finite group of automorphisms of R. If R^G is a Noetherian ring then so is R.*

There are a number of examples showing the necessity of the condition that R be reduced. The following is from Montgomery[28].

EXAMPLE 4. Let K be a field of characteristic different from 2. Set $R = K[x_1, x_2, \ldots]/(x_i x_j) = K[x_1, x_2, \ldots]$, where $x_1, x_2, \ldots$ are indeterminates, $i \leq j$. Let g be the automorphism of R fixing K satisfying $g(x_i) = -x_i$. Set $G = \langle g \rangle$, then $o(G) = 2$ and $R^G = K$, but R is not Noetherian.

Regarding ascent of coherence one obtains the following result:

THEOREM 4. *Let R be a reduced ring, let G be a group of automorphisms of R and assume that R is a finitely generated R^G module. If R^G is a coherent ring then so is R in the following cases:*

1. *G is a finite group and R is o(G)-torsion free.*
2. *G is a locally finite group and R^G is a domain.*
3. *G is a locally finite group, R is semilocal and principal ideals of R^G are flat.*

As this result has not been published elsewhere we sketch the proof here:

1. Let $a_1, \ldots, a_n$ be a set of generators for R over R^G. Define a map $T: R \to \oplus R^G$, one copy of R^G for each generator, by $T(a) = (tr(aa_1), \ldots, tr(aa_n))$, for each a in R, (for a in R $tr(a) = \Sigma\, g(a)$, where the sum is taken over all g in G). T is a 1:1 R^G homomorphism and, via T, R becomes a finitely generated submodule of a coherent R^G module. It follows that R is a coherent module and thus a coherent ring.
2. For a in R denote by $\sigma_1(a) = \Sigma\, y, \ldots, \sigma_n(a) = \Pi\, y$, where y runs over the elements of the orbit of a, the elementary symmetric functions of a. Let x be a nonzero element of R^G and assume that $xa = 0$, then $x\sigma_i(a) = 0$ for all i. Since R^G is a domain $\sigma_i(a) = 0$ for all i, and therefore $a^n = \sigma_1(a)a^{n-1} + \ldots + \sigma_n(a) = 0$. Since R is a reduced ring it is a torsion free R^G module. It follows that R can be embedded

in a finitely generated R^G module, and is therefore a coherent ring.

3. Let $P \in Spec(R)$ and set $p = P \cap R^G$. Since principal ideals of R^G are flat R^G_p is a coherent domain, and by an argument similar to that in 2., R_p is a coherent ring. Since R is semilocal R is a coherent ring.

It is not clear yet if one can relax any of the conditions of this theorem and maintain the conclusion. The following example, inspired by Brewer & Rutter[7], shows that the condition that R be reduced cannot be relaxed even in the case that R is a finitely generated R^G module and R^G is a domain.

EXAMPLE 5. Let S be a coherent domain and let M be an S module satisfying:

1. M has no 2-torsion
2. M is a finitely generated but not finitely presented S module.

Let $R = S\alpha M$, the trivial ring extension of S by M, that is the set $S \times M$ with usual addition and with multiplication defined by $(s,m)(s',m') = (ss', sm' + s'm)$. S embeds in R via $s \to (s,0)$ and becomes a finitely generated R module with generators $(1,0), (0,m_1), \ldots, (0,m_n)$, where $m_1, \ldots, m_n$ generate M over S. R is not a reduced ring as $(0,m)^2 = (0,0)$. Define an automorphism g of R by $g(s,m) = (s,-m)$ for all s in S and m in M, and let $G = \langle g \rangle$. Then $o(G) = 2$ and $R^G = S$ is a coherent domain, but R itself is not a coherent ring since M is not a coherent S module (Glaz[16]).

For a specific example one can use $S = Q[x_1, x_2, \ldots]$, where $x_1, x_2, \ldots$ are indeterminates over Q, and $M = S/(x_i x_j)$, $i \le j$. Note that with this choice of S and M we even have that $o(G) = 2$ is a unit in R.

3. Regularity and finite weak dimension

We now consider the property of coherent regularity and related behavior of homological dimensions.

A ring R is called regular if every finitely generated ideal of R has finite projective dimension.

If R is a Noetherian ring this formulation of regularity coincides with the classical definition. The extension of the notion of regularity from Noetherian to coherent rings went through several stages. In 1971 Bertin[6] defined regularity for local coherent rings, Vasconcelos[35], in 1976, dropped the "local" condition from the definition, Glaz[13], in 1987, separated the finiteness condition of coherence from the homological condition on the ideals of the ring. Bertin, Quentel, Vasconcelos, Glaz and others proved a variety of interesting results for coherent regular rings. A detailed descriptions of many of these results can be found in Glaz[16], and Glaz[18].

Every coherent ring of finite weak dimension is a regular ring. In particular all the classical non Noetherian rings are coherent regular rings. To summarize the situation:

1. R is Von Neumann regular $\Leftrightarrow$ (coherent) of w.dim $R = 0$
2. R is semihereditary $\Leftrightarrow$ R is coherent and w.dim $R \leq 1$
3. R is hereditary $\Leftrightarrow$ (coherent) of gl.dim $R \leq 1$

The set up $R^G \subseteq R$ considered in this paper falls under the following, more general, framework of investigation:

Let $A \subseteq B$ be two rings. Under what conditions does this extension descend or ascend coherent regularity? How do the weak (or global) dimensions of A and B compare? In particular, when does this extension descend or ascend Von Neumann regularity, semihereditarity or hereditarity?

My work in the last couple of years revolved around this general set up for a variety of rings A and B, see Glaz[13 - 20]). With $A = R^G$ and $B = R$, the previous section of this paper describes the results obtained in the first step of the investigation into descent or ascent of coherence regularity, namely the step concerned with descent and ascent of coherence. We will now introduce into the picture the homological property of regularity.

We start by considering descent of coherent regularity in cases of "small" homological dimensions, that is descent of Von Neumann regularity, semihereditarity and hereditarity. These cases were settled by Bergman[4] in 1971 and by Jondrup[24] in 1974. The following is a minor generalization of their results:

THEOREM 5 (Bergman[4], Jondrup[24]). *Let R be a ring and let G be a group of automorphisms of R then:*

1. *If R is Von Neumann regular, so is* R^G.
2. *If R is semihereditary (respectively hereditary) and G is either a locally finite group or* R^G *is a module retract of R, then* R^G *is semihereditary (respectively hereditary).*

We remark that if R is a faithfully flat R^G module then one can easily check that R^G inherits from R Von Neumann regularity, semihereditarity, hereditarity and almost everything else. In Glaz[19] it is shown that the underlying reason for the ease of descent in the cases described in Theorem 5 is the faithful flatness of R over R^G. This can be summarized as follows:

THEOREM 6 (Glaz[19]). *Let R be a ring and let G be a group of automorphisms of R:*

1. *If R^G is Von Neumann regular, then R is a faithfully flat R^G module.*
2. *If R^G is semihereditary, G is a locally finite group and R is reduced, then R is a faithfully flat R^G module.*
3. *If R^G is semihereditary and principal ideals of R are flat, then R is a flat R^G module. If, in addition, R^G is a module retract of R, then R is a faithfully flat R^G module.*

Given the prevailing faithful flatness in cases where R^G has small weak dimension one is tempted to ask whether R is not, in fact, projective over R^G in all these cases. Jondrup[25] provides an example showing that this might not be the case even if R^G is Von Neumann regular. This is, nevertheless, true for many cases (Jondrup[25], Glaz[19]). Other works computing projective dimensions in the set up $R^G \subseteq R$ are Lorenz[26] and Popov[31].

Ascent of Von Neumann regularity, semihereditarity and hereditarity have not yet been considered for our set up. That this will not easily occur, even in case of Von Neumann regularity, one can see by considering Example 4.

Considering descent of coherence regularity for rings of larger - finite or infinite - homological dimensions one first notices that R^G might not inherit regularity from R even if R and R^G are both Noetherian, R is a finitely generated R^G module and R^G is a module retract of R.

EXAMPLE 6. Let K be a field of characteristic 0 and let $R = K[x,y]$ for indeterminates x and y. Let g be the automorphism of R fixing K and satisfying $g(x) = -x$, $g(y) = -y$. Set $G = \langle g \rangle$, then $o(G) = 2$ which is a unit in R so that R^G is a module retract of R. $R^G = K[x^2, xy, y^2]$ which is a non regular Noetherian ring. Moreover one can show (Glaz[19]), that the projective dimension of R over R^G is infinite .

This extension fails to descend regularity precisely because the projective dimension of R over R^G is infinite.

THEOREM 7 (Glaz[19]). *Let R be a ring and let G be a group of automorphisms of R. Assume that R^G is a module retract of R and that the projective dimension of R over R^G is finite. If R is a regular ring then so is R^G. If, in addition, R is a coherent ring then*

$$w.dim\ R^G \leq w.dim\ R + proj.dim_{R^G}\ R$$

COROLLARY 8 (Glaz[19]). *Let R be a ring and let G be a group of automorphisms of R. If either R is a faithfully flat R^G module, or R^G is a module retract of R and R is a finitely generated R^G module of finite projective dimension, then the coherence regularity of R implies that of R^G and*

$$w.dim\ R^G \leq w.dim\ R + proj.dim_{R^G}\ R$$

The finiteness condition on the projective dimension of R over R^G can be slightly relaxed in case R is a $\aleph_0$ Noetherian ring, that is ideals of R are countably generated.

COROLLARY 9 (Glaz[19]). *Let R be an $\aleph_0$ Noetherian coherent regular ring and let G be a group of automorphisms of R. Assume that R^G is a module retract of R and that R is a finitely generated R^G module of finite weak dimension, then R^G is a coherent regular ring and*

$$w.dim\ R^G \leq w.dim\ R + w.dim_{R^G}\ R + 1$$

What is known on descent of regularity in th Noetherian case? Classically this problem was posed in mor concrete terms, namely, R was assumed to be a polynomial rin in n variables over a field K, G a finite group o automorphisms of R fixing K whose order o(G) is not zero i K - so that R^G is a module retract of R - the question the was under what additional conditions will R^G be a polynomia ring over K. In 1967, Serre[33] found necessary and sufficien conditions for this to occur (namely that G be generated b pseudo-reflections).

With this case settled, the question becomes with and G as above, under what additional conditions will R^G b regular. In view of Example 6, one is not likely to be abl to improve on the answer found in Theorem 7 . What on notices in Example 6 is that since R^G is an integrally close domain of Krull dimension two, R^G is a Cohen Macaulay ring The question that can then be asked is to what extent is R likely to inherit Cohen Macaulayness or related propertie from the regularity of R. This is the direction taken by number of investigations carried out under the Noetheria assumption. Several deep and interesting results in thi direction, some of which involve derivations rather the automorphisms, appear in Hochster & Eagon[22] (1971), Hochste & Roberts[23] (1974), Watanabe[37,38] (1974), Stanley[34] (1979), Aramova & Avramov[1] (1986), Aramova[2] (1987).

Hochster & Roberts point out that if R is a Noetheriar regular (Cohen Macaulay) ring and G is a finite group of automorphisms of R whose order is a unit in R^G, then R^G is Cohen Macaulay. This raises an interesting question which I would like to pose as a conjecture.

CONJECTURE. *Let R be a coherent regular ring and let G be a group of automorphisms of R. Assume that R^G is a module retract of R and that R is a finitely generated R^G module, then R^G is a Cohen Macaulay ring.*

I will conclude this paper by saying that the first step toward solving this conjecture is finding the right definition of non Noetherian Cohen Macaulayness. Although non Noetherian depth can be effectively defined (see for example Glaz[16]), the obvious condition equating, locally, depth and Krull dimension might not be the right generalization, since it is not yet known (and it seems difficult to determine) if this equality hold even for a coherent regular ring.

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