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Metamodeling for Variable Annuities

To our students

– Guojun and Emiliano

To my mom Norma and my dad Ponso

– Emiliano

Contents

Preface

Variable annuities are life insurance products that offer various types of financial guarantees. Insurance companies that have a large block of variable annuity business face many challenges. For example, guarantees embedded in variable annuity policies demand sophisticated models for pricing, financial reporting, and risk management. In practice, insurance companies rely heavily on Monte Carlo simulation to calculate the fair market values of the guarantees because the guarantees are complicated and no closed-form valuation formulas are available. One drawback of Monte Carlo simulation is that it is extremely time-consuming and often prohibitive to value a large portfolio of variable annuity contracts because every contract needs to be projected over many scenarios for a long time horizon.

This monograph is devoted to metamodeling approaches, which have been proposed recently in the academic literature to address the computational problems associated with the valuation of large variable annuity portfolios. A typical metamodeling approach involves the following four steps:

1. select a small number of representative variable annuity contracts (i.e., experimental design),
2. use Monte Carlo simulation to calculate the fair market values (or other quantities of interest) of the representative contracts,
3. build a metamodel (i.e., a predictive model) based on the representative contracts and their fair market values, and
4. use the metamodel to estimate the fair market value for every contract in the portfolio.

Using metamodeling approaches can significantly reduce the runtime of valuing a large portfolio of variable annuity contracts for the following reasons: first, building a metamodel only requires using the Monte Carlo simulation model to value a small number of representative contracts; second, the metamodel is usually much simpler and more computationally efficient than Monte Carlo simulation.

This book is primarily written for undergraduate students, who study actuarial science, statistics, risk management, and financial mathematics. It is equally useful for practitioners, who work in insurance companies, consulting firms, and banks. The book is also a source of reference for researchers and graduate students with scholarly interest in computational issues related to

variable annuities and other similar insurance products. The methods presented in the book are described in detail and implemented in R, which is a popular language and environment for statistical computing and graphics. Using the R code and datasets that accompany this book, readers can easily replicate the numerical results presented in the book. In addition, readers can modify the R code included in this book for their own use.

This book is divided into three parts. The first part, which consists of Chapters 1, 2, and 3, introduces the computational problems associated with variable annuity valuation, reviews existing approaches, and presents the metamodeling approach in detail. The second part includes Chapters 4, 5, and 6. This part introduces some experimental design methods, which are used to select representative variable annuity contracts. In particular, we describe Latin hypercube sampling, conditional Latin hypercube sampling, and hierarchical k-means for selecting representative policies. The third part includes Chapters 7 to 12 and introduces six metamodels: ordinary kriging, universal kriging, GB2 regression, rank order kriging, linear model with interactions, and tree-based models. These metamodels are predictive models that have been studied in the academic literature for speeding up variable annuity valuation. The dependencies of the chapters are shown in Figure 1. We implement all the experimental design methods and metamodels using a synthetic dataset, which is described in the appendix of this book.

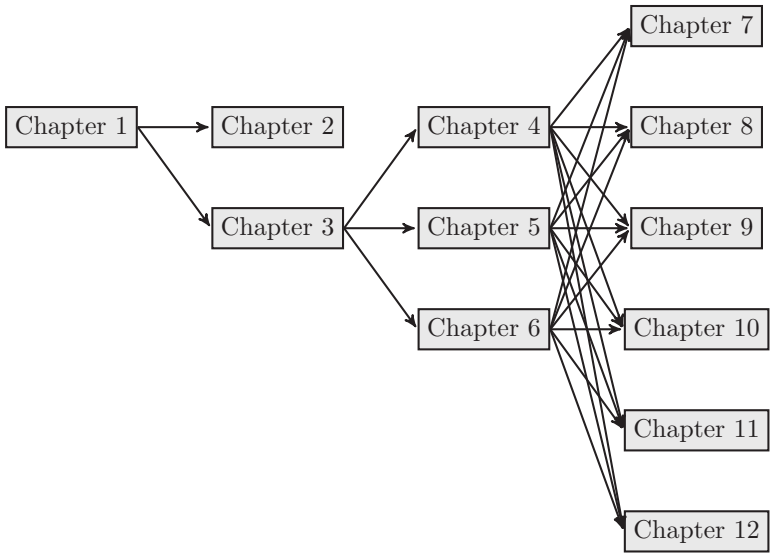

FIGURE 1: Chapter dependencies.

This book has grown from several research projects on variable annuity valuation undertaken by the authors at the University of Connecticut. First,

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