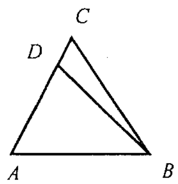
Problems

Problems contain propositions from Book I of Eu-clid's *Elements*. In each instance, prove the indicated result.

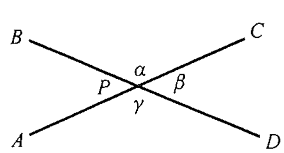
1.

Proposition 6. If two angles of a triangle are congruent with one another, then the sides opposite these angles will also be congruent. [*Hint:* Let ABC be a triangle in which $\angle CAB = \angle CBA$. If $AC \neq BC$, say, $AC > BC$, then choose a point D on AC such that $AD = BC$.]



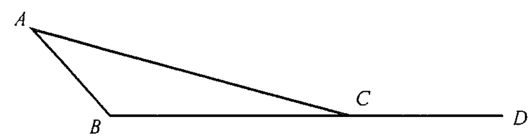
2.

Proposition 15. If two lines cut one another, then they make vertical angles that are equal. [*Hint:* Appeal to Proposition 13, which says that if a ray is drawn from a point on a line, then the sum of the pair of supplementary angles formed is equal to two right angles.]

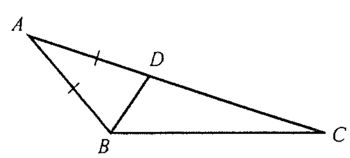


3.

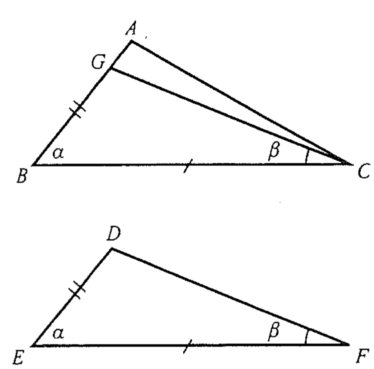
Proposition 17. In a triangle, the sum of any two angles is less than two right angles. [*Hint:* In ΔABC, extend segment BC to a point D and use the exterior angle theorem.]



4. *Proposition 18.* If one side of a triangle is greater than a second side, then the angle opposite the first is greater than the angle opposite the second. [*Hint:* In ΔABC, for $AC > AB$, choose a point D on AC such that $AD = AB$; use the fact that $\angle ADB$ is an exterior angle of ΔBCD.]



5. *Proposition 26.* Two triangles are congruent if they have one side and two adjacent angles of one congruent with a side and two adjacent angles of the other. [*Hint:* Let ΔABC and ΔDEF be such that $\angle B = \angle E$, $\angle C = \angle F$, and $BC = EF$. If $AB \neq DE$, say $AB > DE$, choose a point G on AB for which $BG = ED$.]



Solutions to Euclid Problems

1. Triangles DAB and CBA are congruent by the side-angle-side theorem; hence, $\angle DBA = \angle CAB = \angle CBA$, which contradicts Common Notion 5.

2. $\alpha + \beta = 180^\circ = \beta + \gamma$ implies that $\alpha = \gamma$.

3. Because $\angle ABC < \angle ACD$ by the exterior angle theorem, it follows that $\angle ABC + \angle ACB < \angle ACD + \angle ACB = 180^\circ$.

4. Triangle ABD is isosceles, hence $\angle ABD = \angle ADB$. Applying the exterior angle theorem, $\angle ABC > \angle ABD = \angle ADB > \angle ACB$.

5. Triangles GBC and DEF are congruent by the side-angle-side theorem; hence, $\angle C = \angle F = \angle BCG$, which contradicts Common Notion 5.