1) a) $E = \frac{-p}{10-2p}\cdot(-2)$

$= \frac{2p}{10-2p}$

i) $\frac{2(1)}{10-2(1)} = \frac{2}{8} = \frac{1}{4}$ inelastic

ii) $\frac{2(2.5)}{10-2(2.5)} = \frac{5}{5} = 1$ unit elastic

iii) $\frac{2(4.5)}{10-2(4.5)} = \frac{9}{1} = 9$ elastic

---

b) $E = \frac{-p}{x}\,\frac{dx}{dp}$

$= \frac{-p}{\frac{1}{10+p}}\left(-(10+p)^{-2}\right)$

$= p(10+p)(10+p)^{-2}$

$= p(10+p)^{-1}$

$= \frac{p}{10+p}$

i) $\frac{5}{10+5} = \frac{5}{15} = \frac{1}{3}$ inelastic

ii) $\frac{10}{10+10} = \frac{10}{20} = \frac{1}{2}$ inelastic

iii) $\frac{100}{10+100} = \frac{100}{110} = \frac{10}{11}$ inelastic

---

c) $E = \frac{-p}{x}\,\frac{dx}{dp}$

$= \frac{-p}{10+\frac{1}{p^2}}\cdot\frac{-2}{p^3}$

$= \frac{2p}{10p^3+p}$

$= \frac{2}{10p^2+1}$

i) $\frac{2}{10(0.01)^2+1} = 1.99$ elastic

ii) $\frac{2}{10(1)^2+1} = \frac{2}{11}$ inelastic

2

a) $f'(x) = 2x - 4 = 0$

$x = 2$

$(-\infty, 2)$ $(2, \infty)$
$f' < 0$ $f' > 0$
dec inc

rel min at $f(2) = -3$

b) $f'(x) = 3x^2 + 3 = 0$

none

$(-\infty, \infty)$
$f' > 0$
inc

no relative extrema

c) $f'(x) = -5 - 5x^4 = 0$

$-5(1 + x^4) = 0$

none

$(-\infty, \infty)$
$f' < 0$
dec

no relative extrema

d) $f'(x) = \frac{2}{3}(x+3)^{-1/3}$

$\frac{2}{3(x+3)^{1/3}} = 0$ none

$f'(x)$ DNE at $x = -3$

$(-\infty, -3)$ $(-3, \infty)$
$f' < 0$ $f' > 0$
dec inc

rel min at $f(-3) = 0$

e) $f'(x) = 24x^2 - 48x + 18 = 0$

$6(4x^2 - 4x + 3) = 0$

none

$(-\infty, \infty)$
$f' > 0$
inc

no relative extrema

f) $f'(x) = 20 - 5x^4 = 0$

$5(4 - x^4) = 0$

$x = \pm \sqrt[4]{4} = \pm \sqrt{2}$

$(-\infty, -\sqrt{2})$ $(-\sqrt{2}, \sqrt{2})$ $(\sqrt{2}, \infty)$
$f' < 0$ $f' > 0$ $f' < 0$
dec inc dec

rel min at $f(-\sqrt{2}) \approx -28.94$

rel max at $f(\sqrt{2}) \approx 38.94$

g) $f'(x) = 1 - \frac{1}{x} = 0$

$1 = \frac{1}{x}$

$x = 1$

$f'(x)$ DNE at $x = 0$

$(0, 1)$ $(1, \infty)$

$f' < 0$ $f' > 0$

dec inc

rel min at $f(1) = 1$

---

h) $f'(x) = \dfrac{-2xe^{x^2}}{(1+e^{x^2})^2} = 0$

$-2xe^{x^2} = 0$

$-2x = 0$ $e^{x^2} = 0$

$x = 0$ X

$(-\infty, 0)$ $(0, \infty)$

$f' > 0$ $f' < 0$

inc dec

rel max at $f(0) = \frac{1}{2}$

---

i) $f'(x) = e^{-x} - xe^{-x} = 0$

$e^{-x}(1-x) = 0$

$e^{-x} = 0$ $1 - x = 0$

X $x = 1$

$(-\infty, 1)$ $(1, \infty)$

$f' > 0$ $f' < 0$

inc dec

rel max at $f(1) = \frac{1}{e}$

3)

a) $f'(x) = 25x^4 + 16x^3 - 9x^2 - 4x + 1$

$f''(x) = 100x^3 + 48x^2 - 18x - 4$

---

b) $f(x) = (x+1)^{-1}$

$f'(x) = (-1)(x+1)^{-2} = -(x+1)^{-2} = \dfrac{-1}{(x+1)^2}$

$f''(x) = (-2) \cdot -(x+1)^{-3} = 2(x+1)^{-3} = \dfrac{2}{(x+1)^3}$

---

c) $f(x) = (2x+1)^{1/2}$

$f'(x) = \frac{1}{2}(2x+1)^{-1/2}(2) = (2x+1)^{-1/2}$

$f''(x) = -\frac{1}{2}(2x+1)^{-3/2}(2) = -(2x+1)^{-3/2}$

---

d) $f'(x) = \ln(x) + x\left(\frac{1}{x}\right) = \ln(x) + 1$

$f''(x) = \frac{1}{x}$

---

e) $f'(x) = 2xe^x + x^2e^x = e^x(2x + x^2)$

$f''(x) = e^x(2x + x^2) + e^x(2 + 2x) = e^x(x^2 + 4x + 2)$

---

f) $f'(x) = \dfrac{6x+3}{3x^2+3x-1}$

$$f''(x) = \frac{6(3x^2+3x-1)-(6x+3)(6x+3)}{(3x^2+3x-1)^2}$$

$$= \frac{-18x^2-14x-15}{(3x^2+3x-1)^2}$$

4)

a) $f''(x) = 6x - 6 = 0$

$x = 1$

$(-\infty, 1)$ $(1, \infty)$

$f'' < 0$ $f'' > 0$

CD CU

inf point at $f(1) = 0$

---

b) $f''(x) = 42x^4(6x-5) = 0$

$x = 0$ $x = \frac{5}{6}$

$(-\infty, 0)$ $(0, \frac{5}{6})$ $(\frac{5}{6}, \infty)$

$f'' < 0$ $f'' < 0$ $f'' > 0$

CD CD CU

inf point at $f(\frac{5}{6})$

---

c) $f''(x) = e^x(2+x) = 0$

$e^x = 0$ $2 + x = 0$

X $x = -2$

$(-\infty, -2)$ $(-2, \infty)$

$f'' < 0$ $f'' > 0$

CD CU

inf point at $f(-2)$

---

d) $f''(x) = -2 - 12x^2 = 0$

$-2(1 + 6x^2) = 0$

none

$(-\infty, \infty)$

$f'' < 0$

CD

no inf points

---

e) $f''(x) = \frac{2}{x^3} = 0$

none

$(-\infty, 0)$ $(0, \infty)$

$f'' < 0$ $f'' > 0$

CD CU

no inf points since $f(0)$ DNE

$f''(x) = e^x - e^{-x} = 0$

$e^x = e^{-x}$

$x = -x$

$x = 0$

$(-\infty, 0)$ $(0, \infty)$

$f'' < 0$ $f'' > 0$

CD CU

inf point at $f(0)$

5) a) $f'(x)=0$, $f'(x)$ DNE

$x=0, 2, 4$

b) $f'>0 \Rightarrow$ inc $\Rightarrow (0,\infty)$

$f'<0 \Rightarrow$ dec $\Rightarrow (-\infty,0)$

c) rel min at $x=0$

d) f' inc $\Rightarrow$ CU $\Rightarrow (-\infty,2)\,(4,\infty)$

f' dec $\Rightarrow$ CD $\Rightarrow (2,4)$

---

6) $f(x)=x^3+3x^2-9x+1$

① D: $(-\infty,\infty)$

② none

③ none

④ $f(0)=1$

$f'(x)=3x^2+6x-9$

⑤ $3(x^2+2x-3)=0$

$x=-3, 1$

⑥ $(-\infty,-3)$ $(-3,1)$ $(1,\infty)$

$f'>0$ $f'<0$ $f'>0$

inc ↑ dec ↑ inc

⑦ rel max rel min

$f''(x)=6x+6$

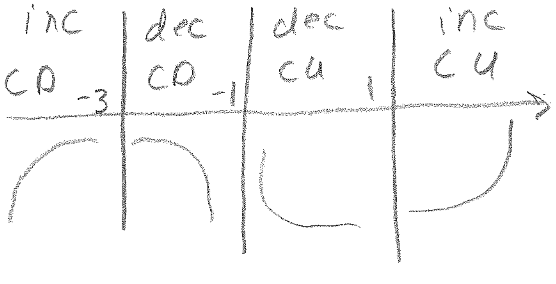
⑧ $6x+6=0$

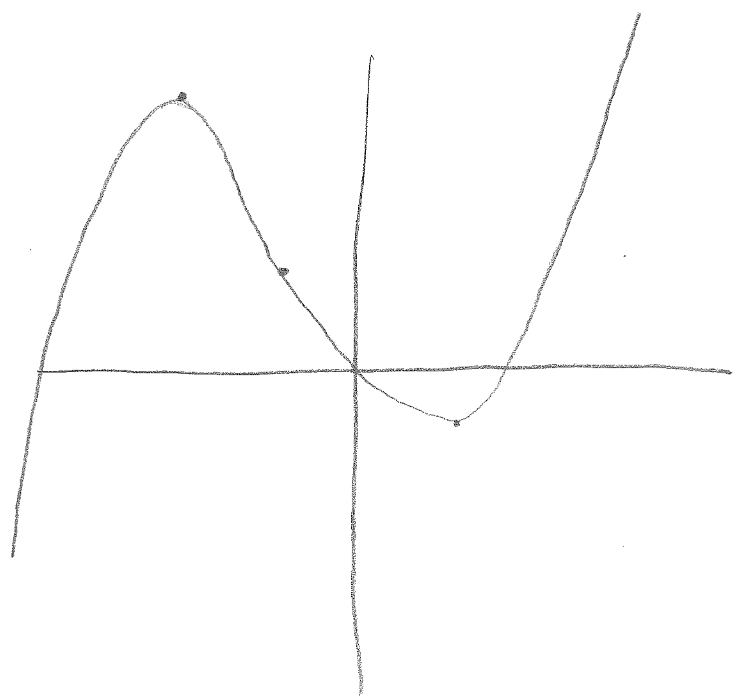
$x=-1$

⑨ $(-\infty,-1)$ $(-1,\infty)$

$f''<0$ $f''>0$

CD CU





$f(-3)=28$

$f(-1)=12$

$f(1)=-4$

7) $f(x) = \frac{5x}{3x-5}$

① D: $x \neq \frac{5}{3}$

② $x = \frac{5}{3}$

③ $\lim_{x \to \pm\infty} f(x) = \frac{5}{3}$

④ $f(0) = 0$

$f'(x) = \frac{-25}{(3x-5)^2}$

⑤ none

⑥ $(-\infty, \frac{5}{3})$ $(\frac{5}{3}, \infty)$

$f' < 0$ $f' < 0$

dec dec

⑦ none

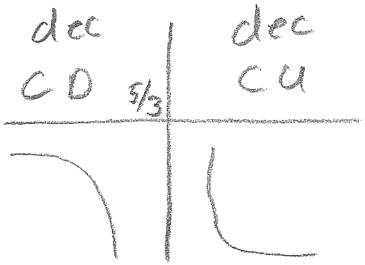
$f''(x) = \frac{150}{(3x-5)^3}$

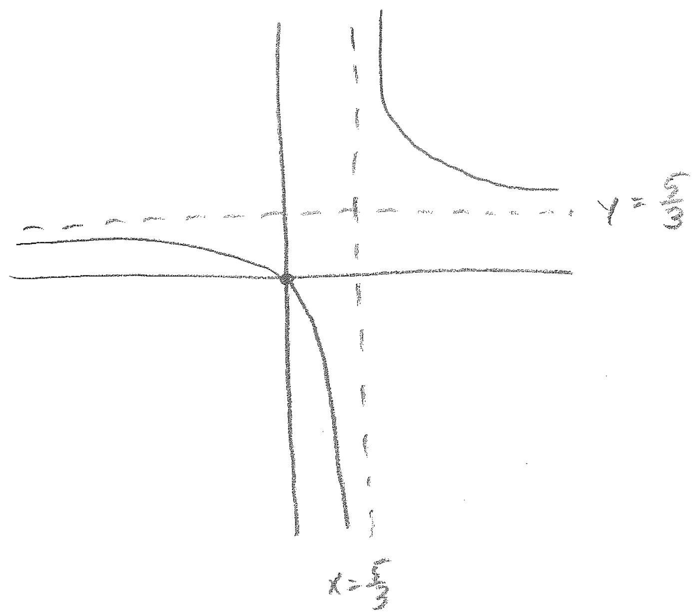
⑧ none

⑨ $(-\infty, \frac{5}{3})$ $(\frac{5}{3}, \infty)$

$f'' < 0$ $f'' > 0$

CD CU





8) D: $(-\infty, \infty)$

no vertical asymptotes

no horizontal asymptotes

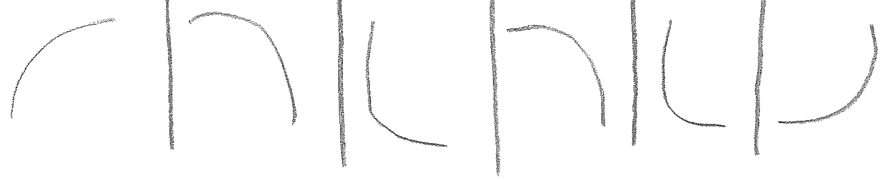
$f' < 0$ $(-3, 3.5)$

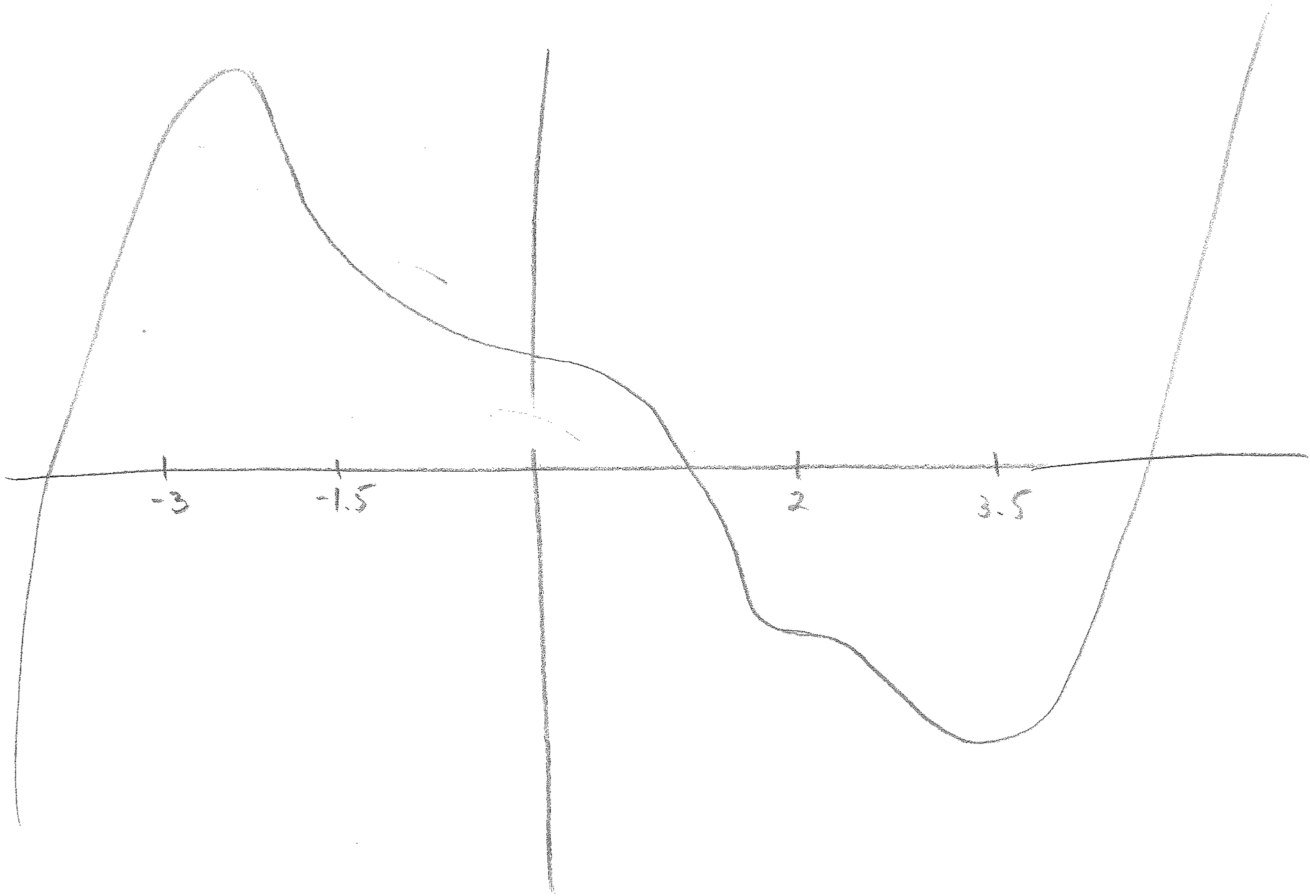
$f' > 0$ $(-\infty, 3)$ $(3.5, \infty)$

$f'' < 0$ $(-\infty, -1.5)$
$(0, 2)$

$f'' > 0$ $(-1.5, 0)$
$(2, \infty)$

inc	dec	dec	dec	dec	inc
CD	CD	CU	CD	CU	CU
-3	-1.5	0	2	3.5	





10)

a) $\lim_{x \to -\infty} 2x^5 - 4x^3 + 1 = \lim_{x \to -\infty} 2x^5 = -\infty$

b) $\lim_{x \to \infty} \frac{2x^2 + x}{x^2 - x + 1} = \lim_{x \to \infty} \frac{2x^2}{x^2} = \lim_{x \to \infty} 2 = 2$

c) $\lim_{x \to -\infty} \frac{3x^3 + x^2 - x + 4}{4x^2 - 1} = \lim_{x \to -\infty} \frac{3x^3}{4x^2} = \lim_{x \to -\infty} \frac{3x}{4} = -\infty$

d) $= \lim_{x \to \infty} \frac{5x^5}{6x^6} = \lim_{x \to \infty} \frac{5}{6x} = 0$

e) $\lim_{x \to \infty} (1 + 2e^{-x}) = 1 + 0 = 1$

f) $\lim_{x \to -\infty} (3 - 2e^{x}) = 3 - 0 = 3$

g) $\lim_{x \to \infty} \frac{10}{5 + e^{-x}} = \frac{10}{5 + 0} = 2$

Use this for Sketching solutions

$f(x)$

1. Domain
2. Vertical Asymptotes
3. Horizontal Asymptotes
4. y-intercept

$f'(x)$

5. Critical Points
6. Inc / Dec
7. Relative Extrema

$f''(x)$

8. Concavity
9. Inflection Points

11)

a) $f(x) = 3x^2 + 24x$

① D: $(-\infty, \infty)$

② none

③ none

④ $f(0) = 0$

⑤ $f'(x) = 6x + 24$

⑤ $x = -4$

⑥ $(-\infty, -4)$ $(-4, \infty)$

$f' < 0$ $f' > 0$

dec inc

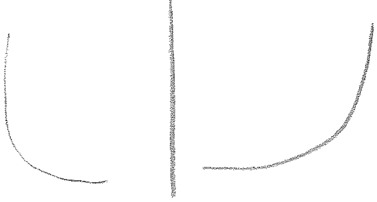
⑦ rel min

$f''(x) = 6 \Rightarrow$ ⑧ CU $(-\infty, \infty)$

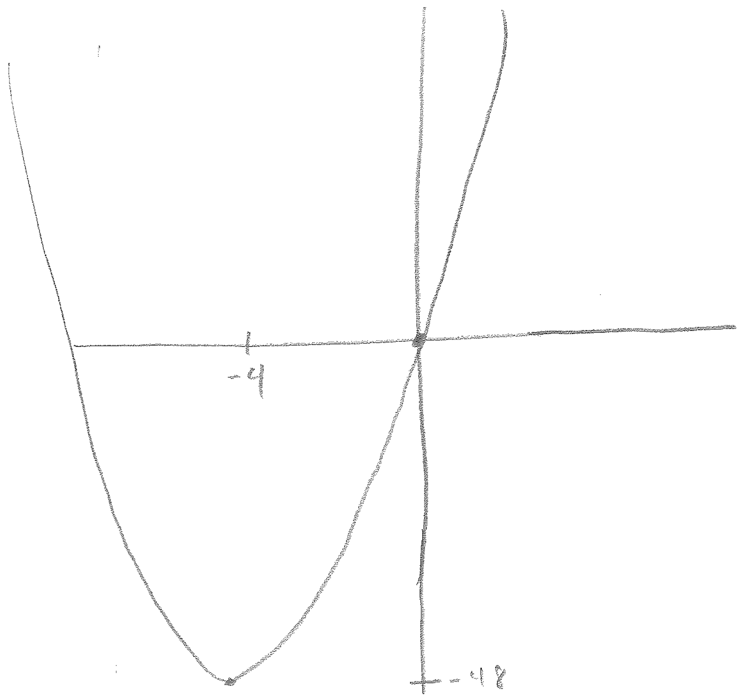
⑨ no inf points

dec	inc
CU	CU

4



$f(-4) =$



b) $f(x) = 2x^3 + 3x^2 - 12x$

① $D: (-\infty, \infty)$

② none

③ none

④ $f(0) = 0$

$f'(x) = 6x^2 + 6x - 12$

⑤ $6(x^2 + x - 2) = 0$

$x = 1 \quad x = -2$

⑥ $(-\infty, -2) \quad (-2, 1) \quad (1, \infty)$

$f' > 0 \quad f' < 0 \quad f' > 0$

inc dec inc

rel max rel min

$f(-2) = 20 \quad f'(1) = -7$

$f''(x) = 12x + 6$

⑦ $12x + 6 = 0$

$x = -\frac{1}{2}$

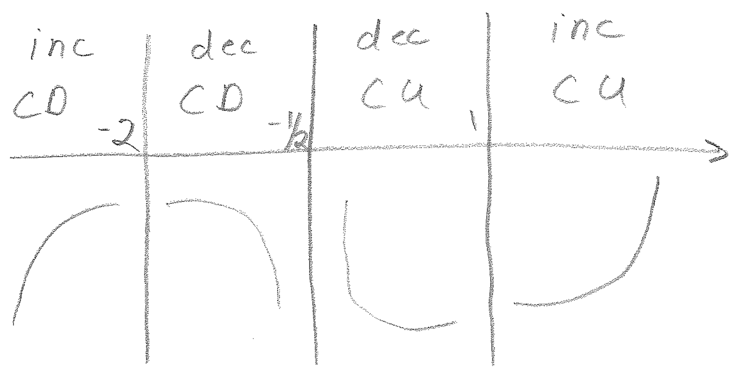
$(-\infty, -\frac{1}{2}) \quad (-\frac{1}{2}, \infty)$

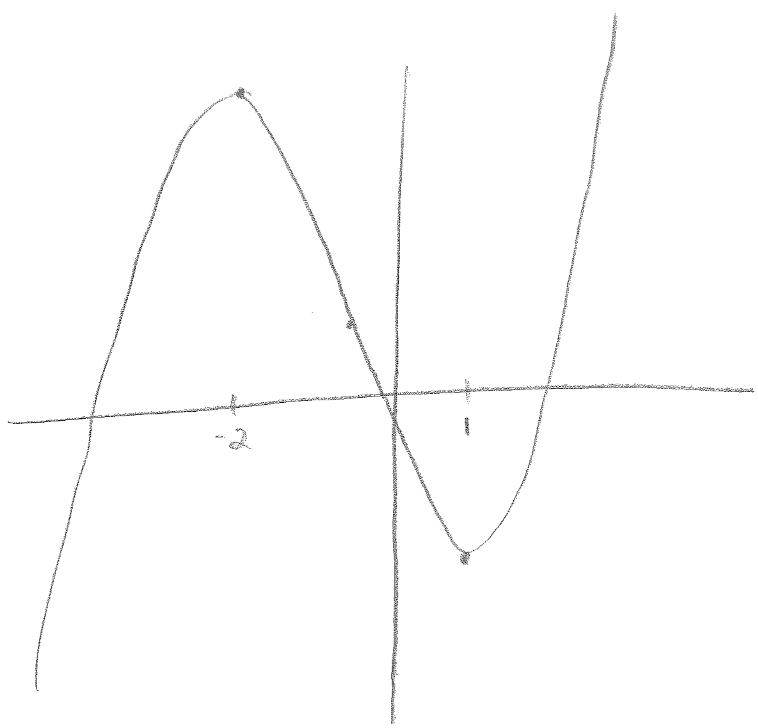
$f'' < 0 \quad f'' > 0$

CD CU

I. P.

$f(-\frac{1}{2}) = 6.5$





c) $f(x) = \frac{8x}{x^2-1}$

① D: $x \neq \pm 1$

② $x = \pm 1$

③ $\lim_{x \to \pm\infty} f(x) = 0$

$y = 0$

④ $f(0) = 0$

$f'(x) = \frac{8(x^2+1)}{(x^2-1)^2}$

⑤ $8(x^2+1) = 0$

X

none

⑥ $(-\infty, -1)$ $(-1, 1)$ $(1, \infty)$

$f'>0$ $f'>0$ $f'>0$

inc inc inc

⑦ no relative extrema

$f''(x) = \frac{16x(x^2+3)}{(x^2-1)^3}$

⑧ $16x(x^2+3) = 0$

$x = 0$

$(-\infty, -1)$ $(-1, 0)$ $(0, 1)$ $(1, \infty)$

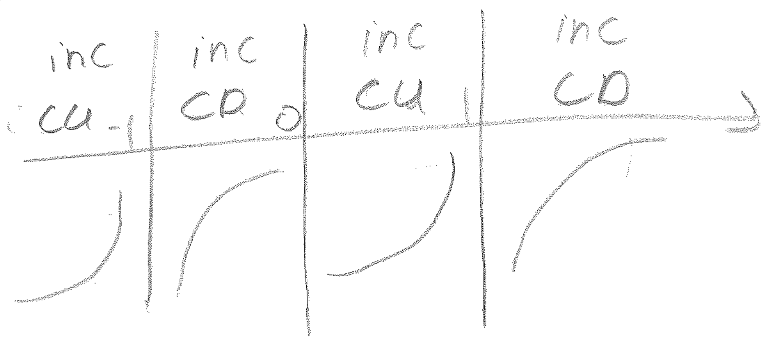
$f''>0$ $f''<0$ $f''>0$ $f''<0$

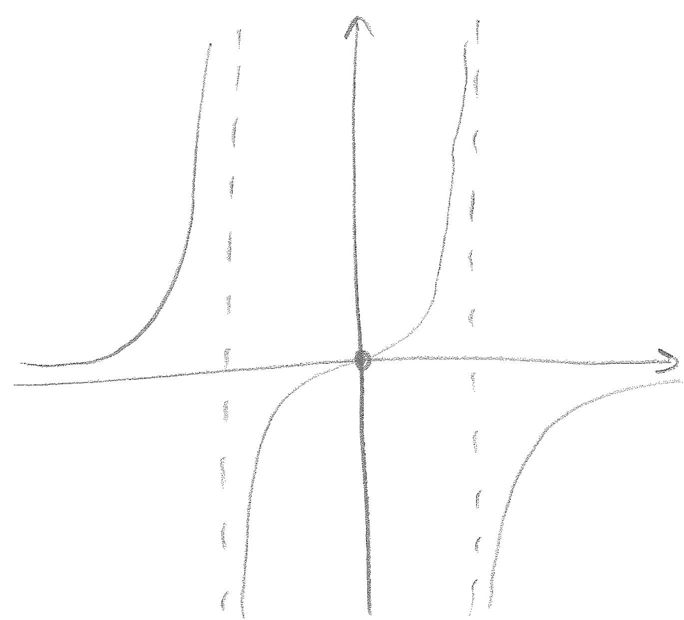
CU CD CU CD

↑

inf pt

$f(0) = 0$





d) $f(x) = \frac{x}{x-1}$

① D: $x \neq 1$

② $x = 1$

③ $\lim_{x \to \pm\infty} f(x) = 1$

$y = 1$

④ $f(0) = 0$

$f'(x) = \frac{-1}{(x-1)^2}$

⑤ none

⑥ $(-\infty, 1)$ $(1, \infty)$

$f' < 0$ $f' < 0$

dec dec

⑦ none

$f''(x) = \frac{2}{(x-1)^3}$

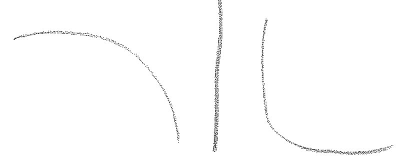
⑧ $(-\infty, 1)$ $(1, \infty)$

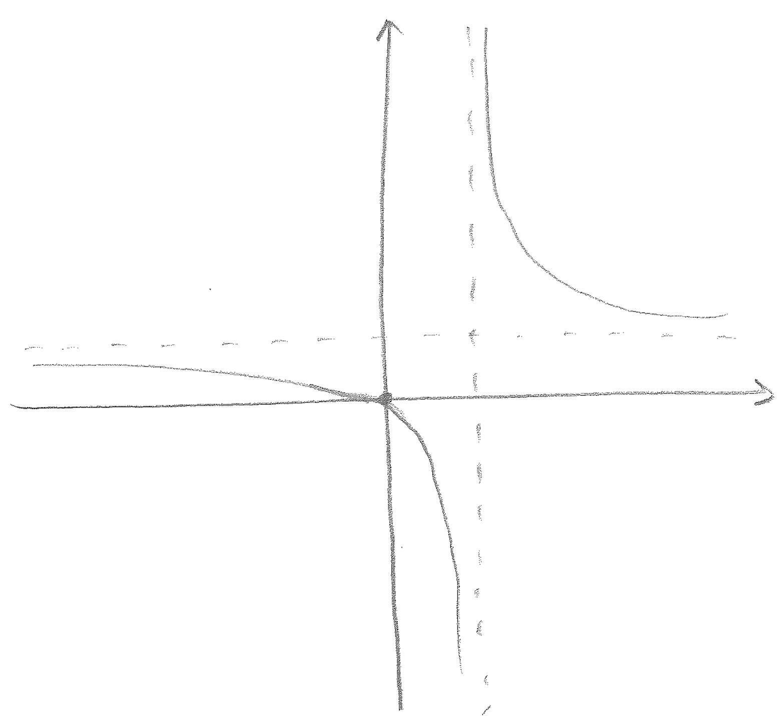
$f'' < 0$ $f'' > 0$

CD CU

⑨ none

dec	dec
CD	CU





e) $f(x) = \dfrac{x^2 + 2x - 4}{x^2}$

① D: $x \neq 0$

② $x = 0$

③ $\lim\limits_{x \to \pm\infty} f(x) = 1$

$y = 1$

④ none

$f''(x) = \dfrac{8 - 2x}{x^3}$

⑤ $8 - 2x = 0$

$x = 4$

⑥ $(-\infty, 0)$ $(0, 4)$ $(4, \infty)$

$f' < 0$ $f' > 0$ $f' < 0$

dec inc dec

rel min

⑦ $f(4) = \frac{5}{4}$

$f''(x) = \dfrac{4(x-6)}{x^4}$

⑧ $x - 6 = 0$

$x = 6$

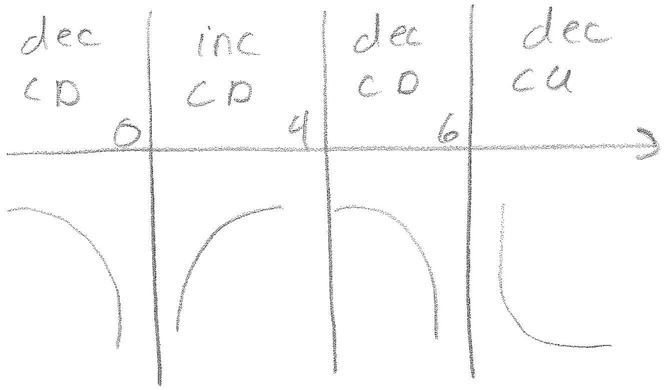
$(-\infty, 0)$ $(0, 6)$ $(6, \infty)$

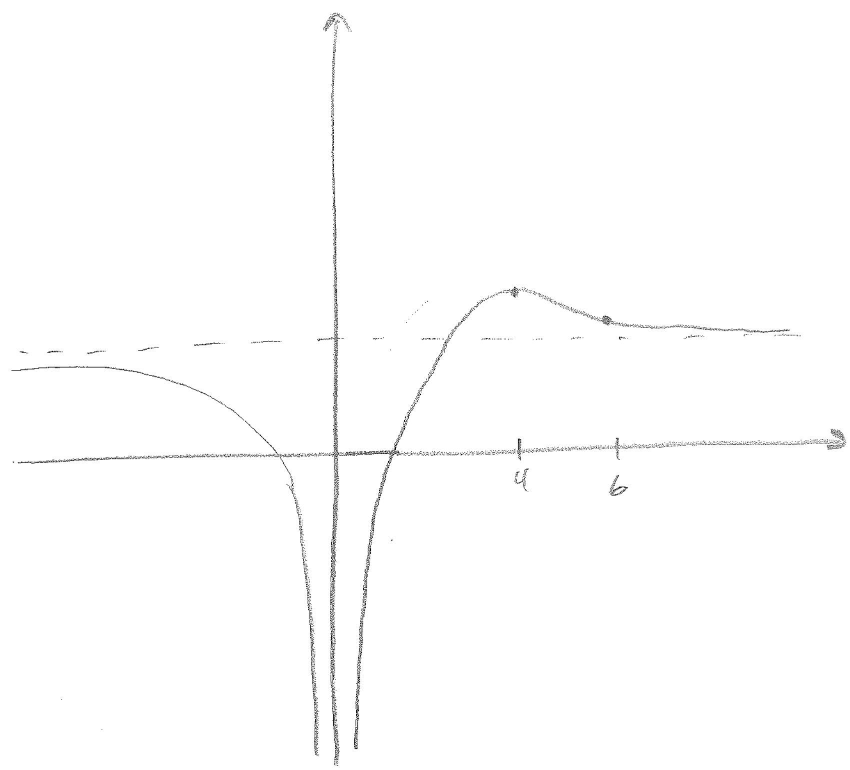
$f'' < 0$ $f'' < 0$ $f'' > 0$

CD CD CU

inf pt

⑨ $f(6) = \frac{11}{9}$





f) $f(x) = x\ln(x)$

① D: $(0, \infty)$

② $x = 0$

③ none

④ none

$f'(x) = \ln(x) + 1$

⑤ $\ln(x) + 1 = 0$

$\ln(x) = -1$

$x = e^{-1} \approx 0.367$

⑥ $(0, e^{-1})$ (e^{-1}, ∞)

$f' < 0$ $f' > 0$

dec inc

⑦ rel min

$f(e^{-1}) = -e^{-1}$

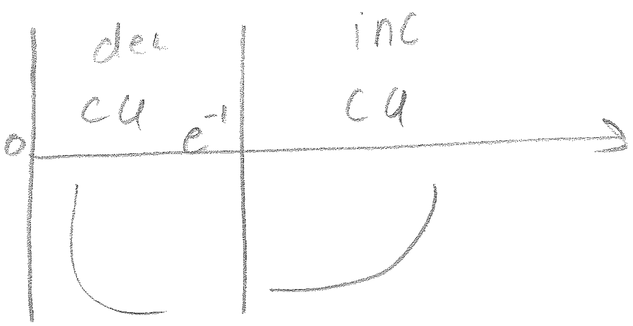
$f''(x) = \frac{1}{x}$

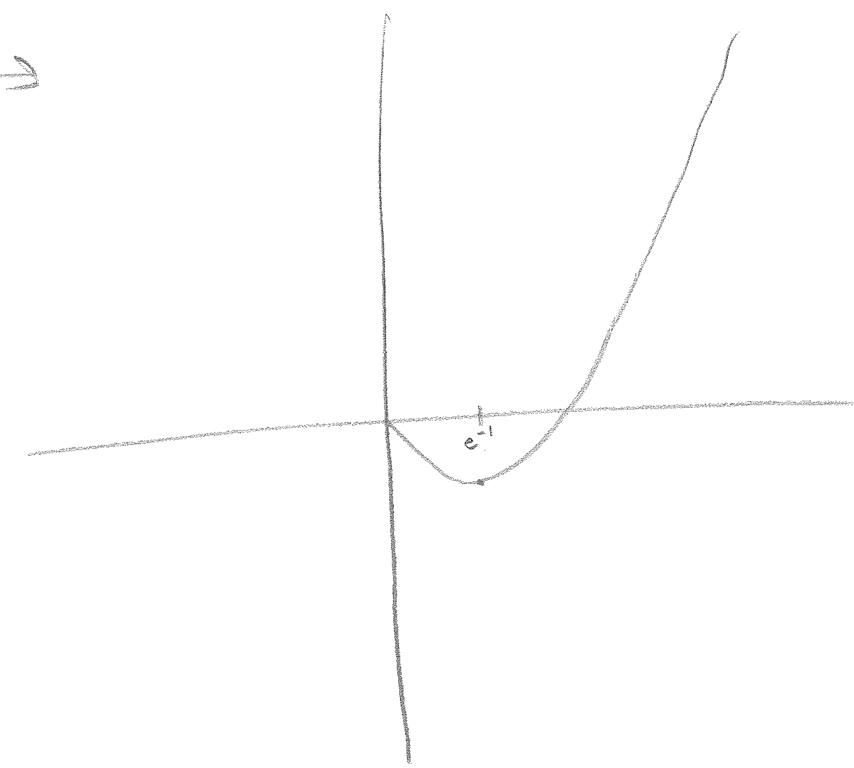
⑧ $(0, \infty)$

$f'' > 0$

CU

⑨ none





12)

a) $f'(x) = 2x - 4 = 0$

$x = 2$

not in $[-1, 1]$

$f(-1) = 6$ abs max

$f(1) = -2$ abs min

b) $f'(x) = 3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0 \quad x = -2$

$f(-3) = -2$ abs min

$f(2) = 18$ abs max

$f(0) = -2$ abs min

$f(-2) = 2$

c) $f'(x) = 2x - 16x^{-3} = 0$

$2x^4 - 16 = 0$

$2(x^4 - 8) = 0$

$x = \pm\sqrt[4]{8} \approx \pm 1.68$

-1.68 not in $[1, 8]$

$f(1) = 17$

$f(8) = 64.25$ abs max

$f(\sqrt[4]{8}) \approx 8.48$ abs min

d) $f'(x) = 2x + 16x^{-3} = 0$

$2x^4 + 16 = 0$

$2(x^4 + 8) = 0$

none

$f(1) = -15$ abs min

$f(8) = 63.75$ abs max

e) $f'(x) = 3 - \frac{3}{x^4} = 0$

$3 = \frac{3}{x^4}$

$x^4 = 1$

$x = \pm 1$

$x = -1$ not in $(0, \infty)$

$\lim_{x \to 0^+} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

so there will be an abs min at $f(1)$

---

f) $f'(x) = 3 - \frac{3}{x^4} = 0$

$x = \pm 1$

$x = 1$ not in $(-\infty, 0)$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

so there will be an abs max at $f(-1)$

---

g) $f'(x) = 1 + \frac{1}{x^2} = 0$

$1 = -\frac{1}{x^2}$

$x^2 = -1$

none

$\lim_{x \to 0^+} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

no absolute extrema

13) $f(x) = x^3 - 3x^2 + 2$

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

---

① $[-1, 2]$

$f(-1) = -2$
$f(2) = -2$ } abs min
$f(0) = 2$ abs max

② $[0, \infty)$

$\lim_{x \to \infty} f(x) = \infty$ so no abs max

$f(0) = 2$
$f(2) = -2$ abs min

③ $[1, 3]$

$f(1) = 0$
$f(3) = 2$ abs max
$f(2) = -2$ abs min

14) a) $x+y=16$

$P = xy$

$y = 16 - x$

$P = x(16-x)$

$P = 16x - x^2 \quad (0,16)$

$P' = 16 - 2x = 0$

$\boxed{x=8}$

$y = 16-8$

$\boxed{y=8}$

$P'' = -2 \Rightarrow CD$

so $x=8$ is a max

b) $x+y=32$

$S = x^2+y^2$

$y = 32 - x$

$S = x^2 + (32-x)^2 \quad (0,32)$

$S' = 2x + 2(32-x)(-1)$

$S' = 4x - 64 = 0$

$\boxed{x=16}$

$y = 32 - x$

$\boxed{y=16}$

$S'' = 4 \Rightarrow CU$

so $x=16$ is a min

c) $2x+y=30$

$P = xy^2$

$x = 15 - \frac{1}{2}y$

$P = (15 - \frac{1}{2}y)y^2$

$P = 15y^2 - \frac{1}{2}y^3$

$(0,30)$

$P' = 30y - \frac{3}{2}y^2 = 0$

$60y - 3y^2 = 0$

$3y(20-y) = 0$

$\cancel{y=0} \quad \boxed{y=20}$

$x = 15 - \frac{1}{2}(20)$

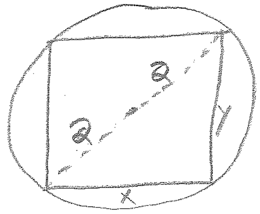
$\boxed{x=5}$

$P'' = 30 - 3y$

$P''(20) < 0 \Rightarrow CD$

so $y=20$ is a max

d)



$x^2+y^2=(4)^2$

$A=xy$

$y^2=16-x^2$

$y=\sqrt{16-x^2}$

$A=x\sqrt{16-x^2} \quad (0,4)$

$A'=\sqrt{16-x^2}+x\left(\frac{1}{2}\right)(16-x^2)^{-1/2}(-2x)$

$A'=\dfrac{16-2x^2}{\sqrt{16-x^2}}=0$

$A'>0$ on $(-2\sqrt{2}, 2\sqrt{2})$

$A'<0$ on $(2\sqrt{2}, 4)$

so $x=2\sqrt{2}$ is a max

$16-2x^2=0$

$x^2=8$

$\boxed{x=2\sqrt{2}}$

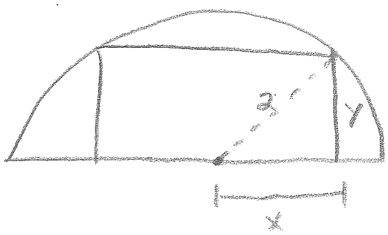
$y=\sqrt{16-(2\sqrt{2})^2}$

$=\sqrt{16-8}$

$=\sqrt{8}$

$\boxed{y=2\sqrt{2}}$

so the rectangle is a square.



$x^2+y^2=4$

$A=2xy$

$y^2=4-x^2$

$y=\sqrt{4-x^2}$

$A=2x\sqrt{4-x^2}$

$$A' = 2\sqrt{4-x^2} + 2x\left(\tfrac{1}{2}\right)(4-x^2)^{-1/2}(-2x)$$

$$A' = \frac{4x^2-8}{\sqrt{4-x^2}} = 0$$

$A' > 0$ on $(-\sqrt{2},\sqrt{2})$

$A' < 0$ on $(\sqrt{2},2)$

so $x=\sqrt{2}$ is a max

$4x^2-8=0$

$x^2=2$

$\boxed{x=\sqrt{2}}$

$y=\sqrt{4-(\sqrt{2})^2}$

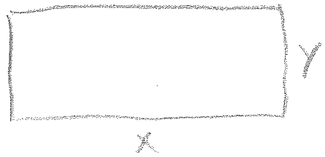
$=\sqrt{4-2}$

$\boxed{y=\sqrt{2}}$

so the dimensions of the rectangle will be $\sqrt{2} \times 2\sqrt{2}$

15)

a)



$2x + 2y = 10$

$x + y = 5$

$A = xy$

$y = 5 - x$

$A = x(5 - x)$

$A = 5x - x^2$

$A' = 5 - 2x = 0$

$\boxed{x = \frac{5}{2}}$

$A'' = -2 \Longrightarrow CD$

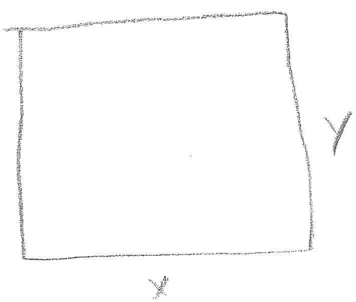
so $x = \frac{5}{x}$ is a max

$y = 5 - \frac{5}{2}$

$\boxed{y = \frac{5}{2}}$

---

b)



$2x + 2y = 400$

$x + y = 200$

$A = xy$

$y = 200 - x$

$A = x(200 - x)$

$A = 200x - x^2$

$A' = 200 - 2x = 0$

$\boxed{x = 100}$

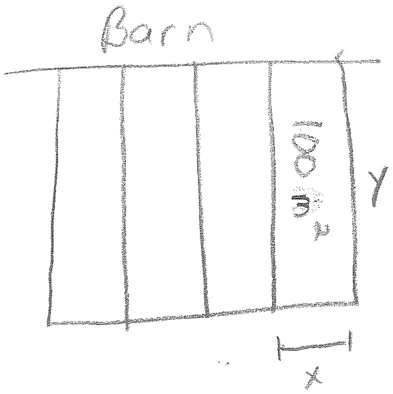
$A'' = -2 \Longrightarrow CD$

so $x = 100$ is a max

$y = 200 - 100$

$\boxed{y = 100}$

c)



$xy = 100$

$P = 5y + 4x$

$x = \frac{100}{y}$

$P = 5y + 4\left(\frac{100}{y}\right)$

$P = 5y + 400y^{-1}$

$(0, \infty)$

$P' = 5 - 400y^{-2} = 0$

$5y^2 - 400 = 0$

$5(y^2 - 80) = 0$

$y^2 = 80$

$\boxed{y = 20\sqrt{2}}$

$P'' = 800y^{-3}$

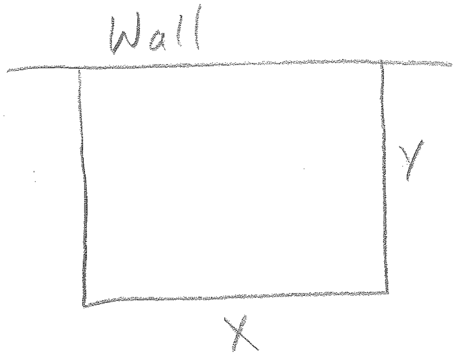
$P''(20\sqrt{2}) > 0 \Longrightarrow CU$

so $y = 20\sqrt{2}$ is a min

$x = \frac{100}{20\sqrt{2}}$

$\boxed{x = \frac{5\sqrt{2}}{2}}$

d)



$xy = 450$

$y = \frac{450}{x}$

$C = 3(2y + x)$

$C = 6y + 3x$

$C = 6\left(\frac{450}{x}\right) + 3x$

$C = 2700x^{-1} + 3x$

$(0, \infty)$

$C' = -2700x^{-2} + 3 = 0$

$-2700 + 3x^2 = 0$

$3(-900 + x^2) = 0$

$x^2 = 900$

$\boxed{x = 30}$

$C'' = 5200x^{-3}$

$C''(30) > 0 \Rightarrow CU$

so $x = 30$ is a min

$y = \frac{450}{3}$

$\boxed{y = 150}$

e) Revenue w/ less than or equal to 1000 trees

$$50 \cdot 1000 = \$50{,}000$$

$R = pn$ let x be the # of trees over 1000

$= (50 - 0.02x)(1000 + x)$ $(0, 2500)$

$R = 50000 + 70x - 0.02x^2$

$R' = 70 - 0.04x = 0$

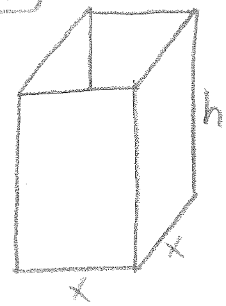
$\boxed{x = 1750}$

$R'' = -0.04 \Rightarrow CD$

so $x = 1750$ is a max

so the orchard should plant 2750 trees

f)



$x^2 h = 27$

$C = 2x^2 + 4xh$

$h = \frac{27}{x^2}$

$C = 2x^2 + 4x\left(\frac{27}{x^2}\right)$

$C = 2x^2 + 108x^{-1}$

$(0, \infty)$

$C' = 4x - 108x^{-2} = 0$

$4x^3 - 108 = 0$

$4(x^3 - 27) = 0$

$\boxed{x = 3}$

$C'' = 4 + 108x^{-3}$

$C''(3) > 0 \Longrightarrow CU$

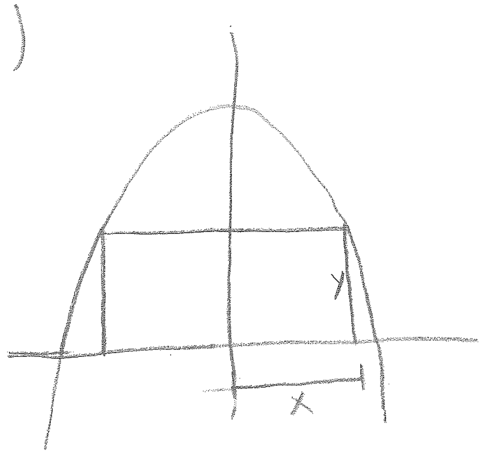
so $x = 3$ is a min

$h = \frac{27}{3^2}$

$\boxed{h = 3}$

so the box is a <u>cube</u>

9)



$y = 16 - x^2$

$A = 2xy$

$A = 2x(16 - x^2)$

$A = 32x - 2x^3$

$A' = 32 - 6x^2 = 0$

$2(16 - 3x^2) = 0$

$\boxed{x = \sqrt{\frac{16}{3}} = \frac{4\sqrt{3}}{3}}$

$A'' = -12x$

$A''\left(\frac{4\sqrt{3}}{3}\right) < 0 \implies CD$

so $x = \frac{4\sqrt{3}}{3}$ is a max

$y = 16 - \left(\frac{4\sqrt{3}}{3}\right)^2$

$= 16 - \frac{16 \cdot 3}{9}$

$= \frac{144 - 72}{9}$

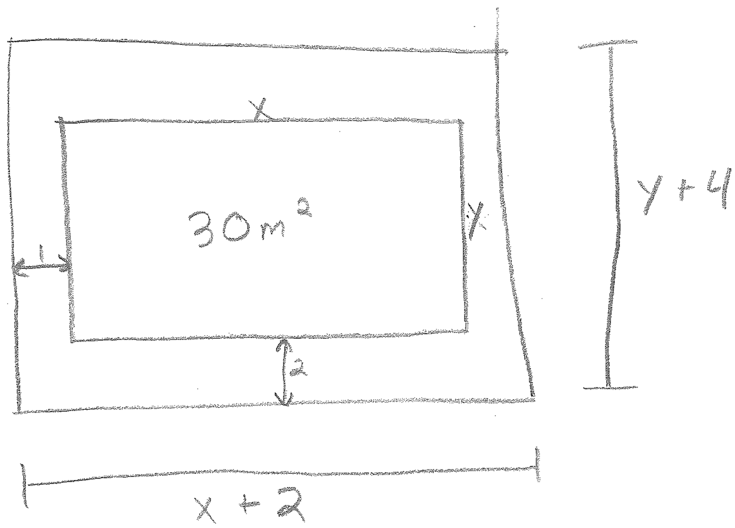
$= \frac{72}{9}$

$\boxed{y = 8}$

So the dimensions of the rectangle are $8 \times \frac{8\sqrt{3}}{3}$

The area is $\frac{64\sqrt{3}}{3}$

h)



$xy = 30$

$A = (x+2)(y+4)$

$y = \dfrac{30}{x}$

$A = (x+2)\left(\dfrac{30}{x} + 4\right)$

$A = 30 + \dfrac{60}{x} + 4x + 8$

$A = 38 + 4x + 60x^{-1}$

$A' = 4 - 60x^{-2} = 0$

$4x^2 - 60 = 0$

$4(x^2 - 15) = 0$

$\boxed{x = \sqrt{15}}$

$A'' = 120x^{-3}$

$A''(\sqrt{15}) > 0 \implies$ CU

so $x = \sqrt{15}$ is a min

$y = \dfrac{30}{\sqrt{15}}$

$\boxed{y = 2\sqrt{15}}$

i) Let x be the # of \$2 increases

$R = pn$

$R = (60+2x)(40-x)$

$R = 2400 + 20x - 2x^2$

$R' = 20 - 4x = 0$

$\boxed{x = 5}$

$R'' = -4 \Rightarrow CD$

so $x = 5$ is a max

So the company should charge \$70 to maximize revenue

16

a) $\int 5\,dx = 5x + C$

---

b) $\int 10(5x+3)^5\,dx$

$u = 5x+3$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$= \int 10u^5 \frac{1}{5}\,du$

$= \int 2u^5\,du$

$= 2\,\frac{1}{6}u^6 + C$

$= \frac{1}{3}(5x+3)^6 + C$

---

c) $\int \frac{1}{2}y^{-3} = \frac{1}{2}\cdot -\frac{1}{2}y^{-2} = -\frac{1}{4}y^{-2} + C$

---

d) $\int (x^2+x+1)\,dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$

---

e) $\int 12x(6x^2+3)^4\,dx$

$u = 6x^2+3$

$du = 12x\,dx$

$= \int u^4\,du$

$= \frac{1}{5}u^5 + C$

$= \frac{1}{5}(6x^2+3)^5 + C$

f) $\int(\sqrt{x} + 2x)\,dx = \frac{2}{3}x^{3/2} + x^2 + C$

g) $\int(1 + 4x^{-2})\,dx = x + 4(-1)x^{-1} + C = x - 4x^{-1}$

h) $\int(5e^t + \frac{7}{t})\,dt = 5e^t + 7\ln(t) + C$

i) $\int 4xe^{-x^2}\,dx$

$u = -x^2$

$du = -2x\,dx$

$-2du = 4x\,dx$

$= \int e^u(-2)\,du$

$= -2\int e^u\,du$

$= -2e^u + C$

$= -2e^{-x^2} + C$

j) $\int \frac{s^5+4}{s^2}\,ds = \int\left(\frac{s^5}{s^2} + \frac{4}{s^2}\right)ds$

$= \int(s^3 + 4s^{-2})\,ds$

$= \frac{1}{4}s^4 + 4(-1)s^{-1} + C$

$= \frac{1}{4}s^4 - 4s^{-1} + C$

k) $\int \frac{1}{t\ln(t^2)}\,dt = \int \frac{1}{t}\cdot\frac{1}{\ln(t^2)}\,dt$

$u = \ln(t^2)$

$du = \frac{2}{t}\,dt$

$\frac{1}{2}\,du = \frac{1}{t}\,dt$

$= \int \frac{1}{u}\left(\frac{1}{2}\right) du$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln(u) + C$

$= \frac{1}{2}\ln\left(\ln(t^2)\right) + C$

---

l) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$

$u = e^x - e^{-x}$

$du = (e^x + e^{-x})\,dx$

$= \int \frac{1}{u}\,du$

$= \ln(u) + C$

$= \ln(e^x - e^{-x}) + C$

m) $\int \frac{x}{\sqrt[4]{4x^2+5}}\, dx$

$u = 4x^2+5$

$du = 8x\ dx$

$\frac{1}{8} du = x\ dx$

$= \int \frac{1}{\sqrt[4]{u}} \left(\frac{1}{8}\right) du$

$= \frac{1}{8} \int u^{-1/4}\ du$

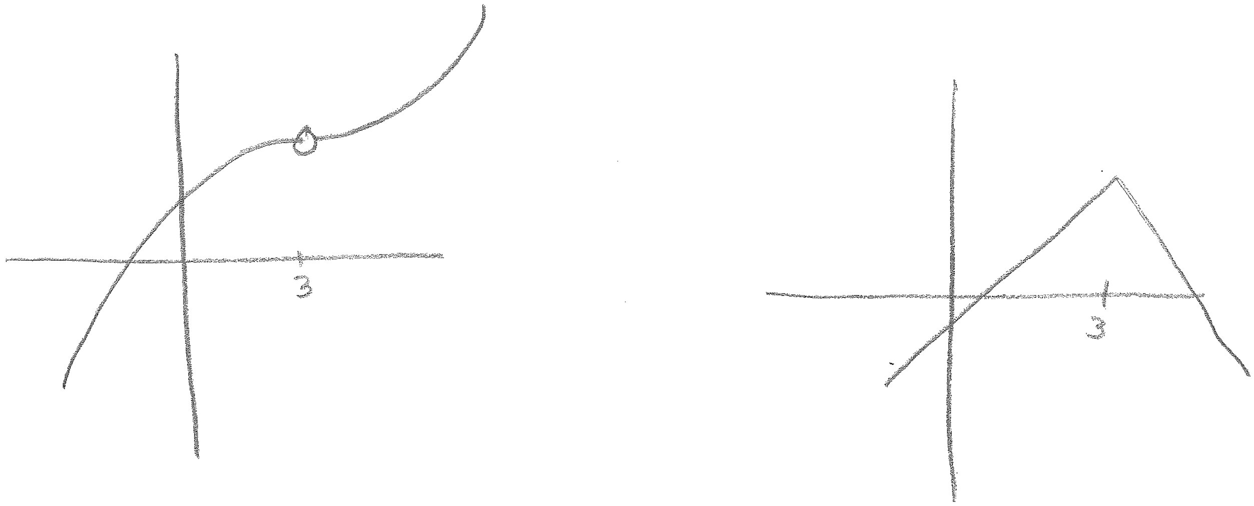
$= \frac{1}{8}\ \frac{4}{3} u^{3/4} + C$

$= \frac{1}{6} u^{3/4} + C$

$= \frac{1}{6} (4x^2+5)^{3/4} + C$

17)

a)

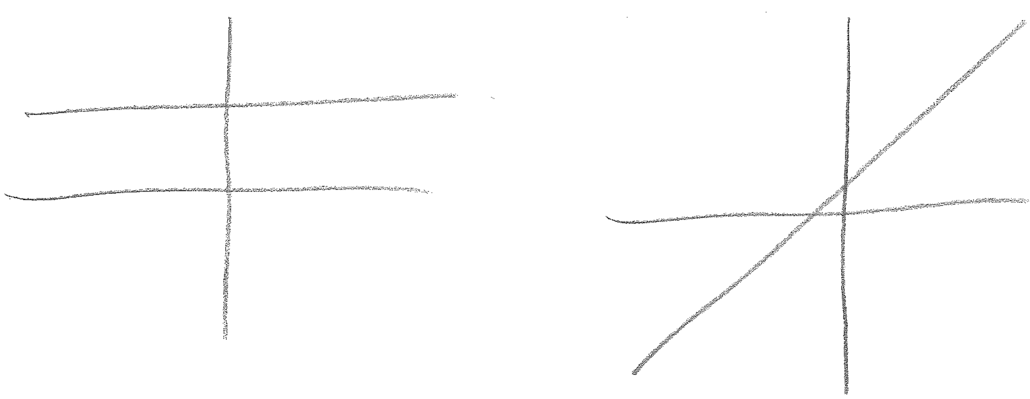


b) $f'(x)$ is increasing. $f''(x)$ tells us where $f'(x)$ is increasing and decreasing.

You can not determine anything about f.

c) They help us determine when a function is increasing and decreasing. We can also determine relative extrema as well.

d)



e) If f' is increasing we know f is concave up.

$f'' > 0$ if f' is increasing

f) It helps us determine if raising the price will increase or decrease revenue. If $E < 1$, inc price will inc revenue and if $E > 1$, then inc price will dec revenue

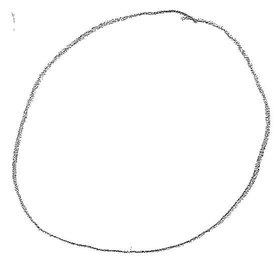
g) Demand is always decreasing so it's derivative, $\frac{dx}{dp}$, will be negative

h) You would find $R'(b) = 0$ and essential perform an absolute extrema proces using the interval $(0, \infty)$.

i) End behavior. To compute horizontal asymptotes and while sketching a graph we would need this.

j) This shows us that that on $(-5, 3)$ f is increasing. Since 1 is also in $(-5, 3)$, $f'(1) > 0$ as well.

k)



$2\pi r = 300$ (constraint)

$A = \pi r^2$ (obj func)

If we had no constraint, the circle could be as large as we want it and there would be no maximum area

l) :)

m) e^x is always concave up so $f'' > 0$

$\ln(x)$ is always concave down so $g'' < 0$

and hence $g'' < f''$ and $g''(100) < f''(100)$