1. Find the elasticity at the given points and determine whether demand is inelastic, elastic, or unit elastic. Explain the significance of your answer.

   (a) $x = 10 - 2p$

       a. $p = 1$ b. $p = 2.5$ c. $p = 4.5$

   (b) $x = \dfrac{1}{10 + p}$

       a. $p = 5$ b. $p = 10$ c. $p = 100$

   (c) $10 + \dfrac{1}{p^2}$

       a. $p = 0.01$ b. $p = 1$

2. Find the intervals where function is increasing and decreasing. Then find all relative extrema.

   (a) $f(x) = x^2 - 4x + 1$

   (b) $f(x) = x^3 + 3x + 1$

   (c) $f(x) = 1 - 5x - x^5$

   (d) $f(x) = (x + 3)^{2/3}$

   (e) $f(x) = 8x^3 - 24x^2 + 18x + 6$

   (f) $f(x) = 5 + 20x - x^5$

   (g) $f(x) = x - \ln x$

   (h) $f(x) = \dfrac{1}{1 + e^{x^2}}$

   (i) $f(x) = xe^{-x}$

3. Find $f''(x)$ of the following functions.

   (a) $f(x) = 5x^5 + 4x^4 - 3x^3 - 2x^2 + x$

   (b) $f(x) = \dfrac{1}{x + 1}$

   (c) $f(x) = \sqrt{2x + 1}$

   (d) $f(x) = x \ln x$

   (e) $f(x) = x^2 e^x$

   (f) $f(x) = \ln(3x^2 + 3x - 1)$

4. Find the intervals of concavity for the following functions and identify any inflection points.

    (a) $f(x) = x^3 - 3x^2 + 2$

    (b) $f(x) = 6x^7 - 7x^6 + 1$

    (c) $f(x) = xe^x$

    (d) $f(x) = 3 - x^2 - x^4$

    (e) $f(x) = \dfrac{1}{x}$

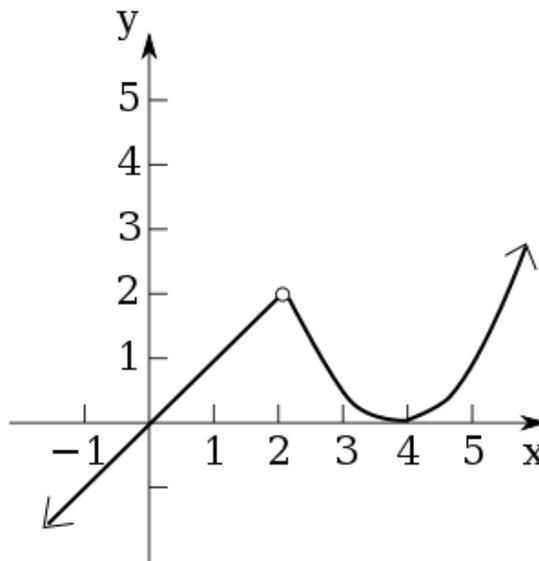   (f) $f(x) = e^x - e^{-x}$

5. The following is a graph of $f'(x)$.

    (a) What are the critical points of $f(x)$?

    (b) Determine the intervals on which $f(x)$ is increasing and decreasing.

    (c) Determine the concavity of $f(x)$.

    (d) Find any relative extrema of f.

6. Let $f(x) = x^3 + 3x^2 - 9x + 1$. Find critical points of f, find the intervals where f is increasing and decreasing, and any relative extrema of f. Find all inflection values, and intervals where f is concave up and concave down. Find if f has any asymptotes. Graph f.
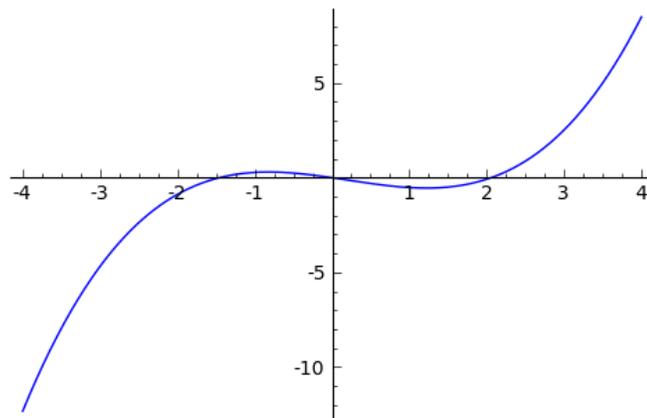
7. Let $f(x) = \frac{5x}{3x-5}$. Find critical points of f, find the intervals where f is increasing and decreasing, and any relative extrema of f. Find all inflection values, and intervals where f is concave up and concave down. Find if f has any asymptotes. Graph f.

8. Say $f'(x)$ and $f''(x)$ have the following graphs. Sketch a graph of $f(x)$.
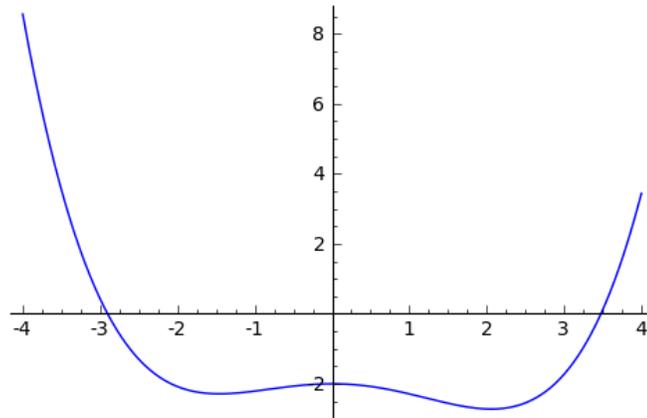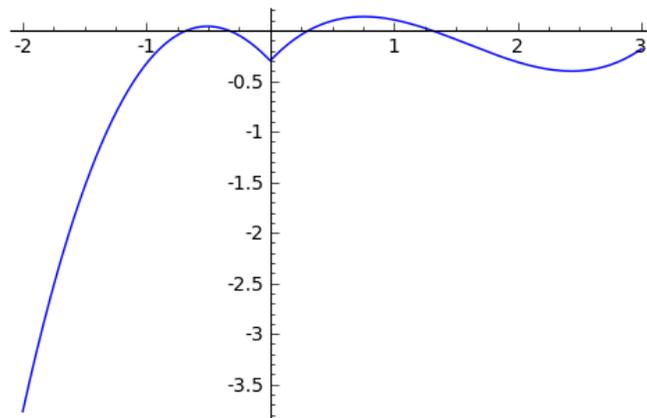
   The graph of $f'(x)$ is:

   The graph of $f''(x)$ is:

9. Let's say the graph of $f(x)$ is given below. Sketch a graph of $f'(x)$. It's a little harder, but also try to sketch a graph of $f''(x)$.

   The graph of $f(x)$ is:

10. Evaluate the limits

   (a) $\lim\limits_{x \to -\infty} 2x^5 - 4x^3 + 1$

   (b) $\lim\limits_{x \to \infty} \dfrac{2x^2 + x}{x^2 - x + 1}$

   (c) $\lim\limits_{x \to -\infty} \dfrac{3x^3 + x^2 - x + 4}{4x^2 - 1}$

   (d) $\lim\limits_{x \to \infty} \dfrac{5x^5 - 4x^4 - x}{6x^6 + 3x^5 - x^2 - x}$

   (e) $\lim\limits_{x \to \infty} (1 + 2e^{-x})$

   (f) $\lim\limits_{x \to -\infty} (3 - 2e^x)$

   (g) $\lim\limits_{x \to \infty} \dfrac{10}{5 + e^{-x}}$

11. Using the procedure outlined in section 5.4, sketch a graph of the following functions.

   (a) $f(x) = 3x^2 + 24x$
   (b) $f(x) = 2x^3 + 3x^2 - 12x$
   (c) $f(x) = \dfrac{8x}{x^2 - 1}$
   (d) $f(x) = \dfrac{x}{x - 1}$
   (e) $f(x) = \dfrac{x^2 + 2x - 4}{x^2}$
   (f) $f(x) = x \ln x$
   (g) $f(x) = xe^{-x^2}$

12. Find the absolute extrema of the following functions.

   (a) $f(x) = x^2 - 4x + 1$ on $[-1, 1]$
   (b) $f(x) = x^3 + 3x^2 - 2$ on $[-3, 2]$
   (c) $f(x) = x^2 + 16x^{-2}$ on $[1, 8]$
   (d) $f(x) = x^2 - 16x^{-2}$ on $[1, 8]$
   (e) $f(x) = 3x + \dfrac{1}{x^3}$ on $(0, \infty)$
   (f) $f(x) = 3x + \dfrac{1}{x^3}$ on $(-\infty, 0)$
   (g) $f(x) = x - \dfrac{1}{x}$ on $(0, \infty)$

(h) $f(x) = x^3 - 9x^2 + 24x + 1$ on $[0, 5]$

(i) $f(x) = \dfrac{\ln x}{x}$ on $[1, e^2]$

13. Find the absolute extrema of $f(x) = x^3 - 3x^2 + 2$ on the following intervals.

    1. $[-1, 2]$

    2. $[0, \infty)$

    3. $[1, 3]$

14. Intro to Optimization

    (a) Find two numbers whose sum is 16 and whose product is maximum.

    (b) Find two numbers whose sum is 32 and for which the sum of the squares is a minimum.

    (c) Find two nonnegative numbers x and y with $2x + y = 30$ for which the term xy^2 is maximized.

    (d) What is the area of the largest rectangle that can be enclosed by a circle of radius a?

    (e) What is the area of the largest rectangle that can be enclosed by a semicircle of radius a?

15. Optimization

    (a) Of all the rectangles with perimeter of 10 meters, which one has the maximum area?

    (b) A farmer with 400 feet of fencing wishes to construct a rectangular pasture with maximum area. What should the dimensions be?

    (c) A rancher plans to make four identical and adjacent rectangular pens against a barn, each with an area of 100 square meters. What are the dimensions of each pen that minimize the amount of fence that must be used.

    (d) A farmer wishes to enclose a rectangular field of area 450 square feet using an existing wall as one of the sides. The cost of the fence for the other three sides is $3 per foot. Find the dimensions of the rectangular field that minimizes the cost of the fence.

    (e) An apple orchard produces annual revenue of $50 per tree when planted with 1000 trees. Because of overcrowding, the annual revenue per tree is reduced by 2 cents for each additional tree planted. If the cost of maintaining each tree is $10 per tree, how many trees should be planted to maximize total profit from the orchard.

    (f) A company wished to design a rectangular box with square base and no top that will have a volume of 27 cubic inches. The cost per square inch of the bottom is twice the cost of the sides. What should the dimensions be to minimize the cost? What is the minimum cost?

    (g) A rectangle is constructed with its base on the x-axis and two of its vertices on the parabola $y = 16 - x^2$. What are the dimensions of the rectangle with maximum area? What is the area?

(h) A rectangular flower garden with an area of 30 square meters is surrounded by a grass border 1 meter wide on two sides and 2 meters wide on the other two sides. What dimensions of the garden minimize the combines area of the garden and borders?

(i) A bus company charged $60 per person for a sightseeing trip and obtained 40 people for the trip. The company has data that indicates that for the same trip, each $2 increase in the price above $60 results in the loss of one customer. What should the company charge to maximize revenue?

16. Find the general antiderivative.

(a) $\int 5\, dx$

(b) $\int 10(5x + 3)^5\, dx$

(c) $\int \frac{1}{2y^3}\, dy$

(d) $\int (x^2 + x + 1)\, dx$

(e) $\int 12x(6x^2 + 3)^4\, dx$

(f) $\int \left(\sqrt{x} + 2x \right) dx$

(g) $\int \left(1 + \frac{4}{x^2} \right) dx$

(h) $\int \left(5e^t + \frac{7}{t} \right) dt$

(i) $\int 4xe^{-x^2}\, dx$

(j) $\int \frac{s^5 + 4}{s^2}\, ds$

(k) $\int \frac{1}{t \ln(t^2)}\, dt$

(l) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$

(m) $\int \frac{x}{\sqrt[4]{4x^2 + 5}}\, dx$

17. Short answer questions. These don't require any algebra, but they do require that you understand the course material.

(a) Draw a function $f(x)$ such that $f'(3)$ does not exist.

(b) If $f''(x) = \frac{1}{4}$, is $f'(x)$ increasing, decreasing, or can you not tell? Is f increasing, decreasing, or can you not tell?

(c) Explain why critical values are helpful in graphing functions.

(d) Draw a function that has no critical values.

(e) If f' is increasing, what does that say about f? What does it say about f''?

(f) Why is it useful to know the price elasticity of demand?

(g) The price elasticity of demand at a point is given by $E = -\frac{p}{x}\frac{dx}{dp}$. The negative sign is there so that E will be positive (because $\frac{dx}{dp}$ is always negative). Why is $\frac{dx}{dp}$ always negative?

(h) Suppose I'm running a soup shop, and I find that my revenue is given by $R(b) = 12b^3 - 15b^2 + 5$, where b is the number of bowls of soup I sell. Explain how I would go about maximizing my revenue (you don't actually have to maximize it).

(i) What are we figuring out when we compute $\lim_{x\to\infty} f(x)$? Describe two situations in which we would want to compute this.

(j) Suppose I find that critical values of a function f are -5 and 3. I calculate that $f'(0) = 7$. How does that help me? Does that tell me anything about $f'(1)$?

(k) What happens if you remove a constraint from an optimization problem? Consider the following problem. "A farmer wishes to build a fence for his albatross. He has 300 feet of fencing, and wants to build a circular pen. Find the maximum area his circular pen can enclose." You don't need to solve this problem. Instead, identify the constraint equation, and explain what would happen if you removed it from the problem.

(l) Make up an optimization problem from your own life. (No, a problem like this won't actually be on the exam. Don't worry. But it can be interesting to think about.)

(m) Say $f(x) = e^x$ and $g(x) = \ln x$. Which is bigger, $f''(100)$ or $g''(100)$? (Hint: you don't need to compute f'' or g''.)