Final Exam Review (Part 2) - Solutions

1) $P(\text{sum not 6}) = 1 - P(\text{sum 6})$

$= 1 - \frac{5}{144}$

$= \frac{144}{144} - \frac{5}{144}$

$= \frac{139}{144}$

sum 6	
d_1	d_2
1	5
2	4
3	3
4	2

2) a.) $_{15}P_8 = \frac{15!}{(15-8)!} = \frac{15!}{7!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = 259,459,200$

b.) $_{10}C_7 = \frac{10!}{7!(10-7)!} = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}\,3!} = \frac{\overset{5}{\cancel{10}} \cdot \overset{3}{\cancel{9}} \cdot 8}{\cancel{3} \cdot 2 \cdot 1} = 5 \cdot 3 \cdot 8 = 120$

3) a.) $P(\text{children or pets}) = P(\text{children}) + P(\text{pets}) - P(\text{children and pets})$

$= \frac{37}{54} + \frac{23}{54} - \frac{9}{54}$

$= \frac{51}{54} = 0.9444 \Rightarrow 94.44\%$

b.) $P(\text{neither children nor pets}) = 1 - P(\text{children or pets})$

$= 1 - \frac{51}{54}$

$= \frac{54}{54} - \frac{51}{54}$

$= \frac{3}{54} = 0.05556 \Rightarrow 5.556\%$

4)

$P(\text{bus}) = \frac{290}{1000} = 0.29 \leq P(f \mid \text{bus}) = 0.48,\ P(m \mid \text{bus}) = 0.52$

$P(\text{hum}) = \frac{70}{1000} = 0.07 \leq P(f \mid \text{hum}) = 0.60,\ P(m \mid \text{hum}) = 0.40$

$P(\text{edu}) = \frac{180}{1000} = 0.18 \leq P(f \mid \text{edu}) = 0.67,\ P(m \mid \text{edu}) = 0.33$

$P(\text{other}) = \frac{460}{1000} = 0.46 \leq P(f \mid \text{other}) = 0.41,\ P(m \mid \text{other}) = 0.59$

a.) $P(\text{hum}) = \frac{70}{1000} = 0.07 \Rightarrow 7\%$

b.) $P(f \text{ and edu}) = P(f \mid \text{edu})\,P(\text{edu})$

$= (0.67)(0.18)$

$= 0.1206 \Rightarrow 12.06\%$

c.) $P(f) = P(f \text{ and bus}) + P(f \text{ and hum}) + P(f \text{ and edu}) + P(f \text{ and other})$

$= P(f \mid \text{bus})\,P(\text{bus}) + P(f \mid \text{hum})\,P(\text{hum}) + P(f \mid \text{edu})\,P(\text{edu}) + P(f \mid \text{other})\,P(\text{other})$

$= (0.48)(0.29) + (0.60)(0.07) + (0.67)(0.18) + (0.41)(0.46)$

$= 0.1392 + 0.042 + 0.1206 + 0.1886$

$= 0.4904$

d.) $P(\text{edu} \mid m) = \frac{P(m \mid \text{edu})\,P(\text{edu})}{P(m)}$

$= \frac{(0.40)(0.18)}{1 - 0.4904}$

$= \frac{0.072}{0.5096}$

$= 0.1413$

e.) $P(f \mid \text{not bus}) = \frac{P(f \text{ and not bus})}{P(\text{not bus})} = \frac{P(f \mid \text{hum})\,P(\text{hum}) + P(f \mid \text{edu})\,P(\text{edu}) + P(f \mid \text{other})\,P(\text{other})}{P(\text{not bus})}$

$= \frac{(0.60)(0.07) + (0.67)(0.18) + (0.41)(0.46)}{1 - 0.29}$

$= \frac{0.042 + 0.1206 + 0.1886}{0.71} = \frac{0.3512}{0.71} = 0.4946 \Rightarrow 49.46\%$

f.) $P(\text{not bus} \mid f) = \frac{P(f \mid \text{not bus})\,P(\text{not bus})}{P(f)}$

$= \frac{(0.4946)(1 - 0.29)}{0.4904} = \frac{(0.4946)(0.71)}{0.4904} = \frac{0.351166}{0.4904} = 0.7161$

5) a.) $15 + 4 + 2 = 21$ total so 21! different ways to [illegible]

b.) $\underbrace{15!}_{\text{campers}} \quad \underbrace{2!}_{\text{mascots}} \quad \underbrace{4!}_{\text{counselors}} = 15! \cdot 2! \cdot 4!$

6) a.) $P(\text{3 same color}) = P(\text{3 red or 3 white or 3 blue})$

$$= \frac{{}_6C_3}{{}_{21}C_3} + \frac{{}_5C_3}{{}_{21}C_3} + \frac{{}_{10}C_3}{{}_{21}C_3}$$

b.) $P(\text{3 diff. color}) = P(\text{1 red and 1 white and 1 blue})$

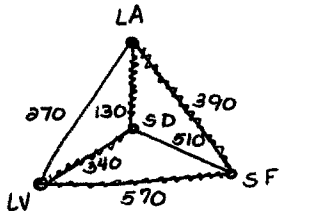
$$= \frac{({}_6C_1)({}_5C_1)({}_{10}C_1)}{{}_{21}C_3}$$

c.) $P(\text{at least 2 are the same color}) = 1 - P(\text{all diff. color})$

$$= 1 - \left(\frac{({}_6C_1)({}_5C_1)({}_{10}C_1)}{{}_{21}C_3} \right)$$

7)

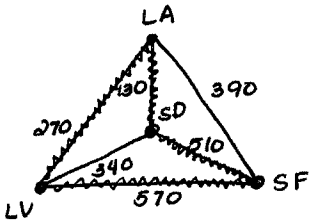
Nearest Neighbor



$LA \xrightarrow{130} SD \xrightarrow{340} LV \xrightarrow{570} SF \xrightarrow{390} LA$

dist = 1430

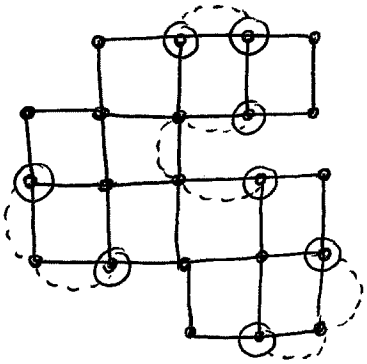
Greedy



$LA \xrightarrow{270} LV \xrightarrow{570} SF \xrightarrow{510} SD \xrightarrow{130} LA$

dist = 1480

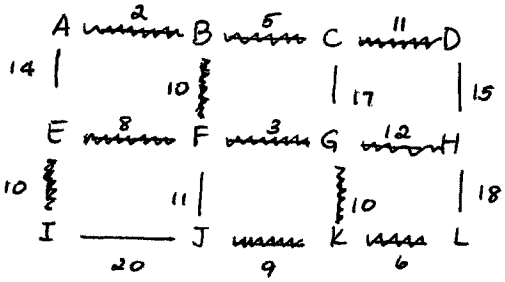
8)



Streets cannot be visited only once because there are more than two odd vertices (there are 8 in fact), hence an eulerian circuit / path cannot be formed

Eulerization is shown with dotted line

9)



dist = 2+5+11+10+8+3+10+10+9+6+12 = 86