

Final Exam Review - SOLUTIONS

① a) $\left. \begin{array}{l} \text{kings} - 11+7=18 \\ \text{Black} - 7+5+9=21 \\ \text{Thievery} - 6+5=11 \\ \text{Ray} - 19 \end{array} \right\} \text{The Black keys wins plurality}$ ✓

b) $\left. \begin{array}{l} \text{Black} - 21+11=32 \\ \text{Ray} - 19+7+6+5=37 \end{array} \right\} \text{Ray LaMontagne wins runoff}$

c) $\left. \begin{array}{l} \text{kings} - 4(18) + 3(9+5+19) + 2(7+6) + 1(5) = 4(18) + 3(33) + 2(13) + 5 = 202 \\ \text{Black} - 4(21) + 3(11) + 2(7+19) + 1(6+5) = 4(21) + 3(11) + 2(26) + 11 = 180 \\ \text{Thievery} - 4(11) + 3(7+5) + 2(9) + 1(11+7+19) = 4(11) + 3(12) + 2(9) + 37 = 135 \\ \text{Ray} - 4(19) + 3(7+6) + 2(11+5+5) + 1(7+9) = 4(19) + 3(13) + 2(21) + 16 = 173 \end{array} \right\}$

kings of Leon wins Borda

d) $\frac{c(c-1)}{2} = \frac{4(3)}{2} = 6$

1.) $\left. \begin{array}{l} \text{kings} - 11+7+6+5+19=48 \\ \text{Black} - 7+5+9=21 \end{array} \right\} \text{kings wins vs. Black}$

2.) $\left. \begin{array}{l} \text{Black} - 11+7+7+5+9+6+19=58 \\ \text{Thievery} - 6+5=11 \end{array} \right\} \text{Black wins vs. Thievery}$

3.) $\left. \begin{array}{l} \text{Thievery} - 7+5+9+6+5=32 \\ \text{Ray} - 11+7+19=37 \end{array} \right\} \text{Ray wins vs. Thievery}$

4.) $\left. \begin{array}{l} \text{kings} - 11+7+9+19=46 \\ \text{Thievery} - 7+5+6+5=23 \end{array} \right\} \text{kings wins vs. Thievery}$

5.) $\left. \begin{array}{l} \text{kings} - 11+7+7+9+5=39 \\ \text{Ray} - 5+6+19=30 \end{array} \right\} \text{kings wins vs. Ray}$

6.) $\left. \begin{array}{l} \text{Black} - 11+7+5+9=32 \\ \text{Ray} - 7+6+5+19=37 \end{array} \right\} \text{Ray wins vs. Black}$

kings of Leon is the Condorcet

e)	original rankings	modified	New Borda counts
	kings - 2	→ 3	$202 - (3 \times 19) + (2 \times 19) = 183$
	Black - 3	→ 4	$180 - (2 \times 19) + (1 \times 19) = 161$
	Thievery - 4	→ 2	$135 - (1 \times 19) + (3 \times 19) = 173$
	Ray - 1	→ 1	173 so strategic voting <u>NOT</u> possible

2) a.) Hamilton's Method, natural divisor = $118,284 \div 12 = 9857$

Store	Shoppers	Nat Quota	In. Alloc	Fin. Alloc	
Boston	28,772	2.9189 ✓	2	3	
Mansfield	13,472	1.3667	1	1	
Providence	32,871	3.3348	3	3	(* final allocation based on largest decimal of natural quota)
Hartford	43,169	4.3795 ✓	4	5	
<u>totals:</u>	<u>118,284</u>		<u>10</u>	<u>12</u>	

b.) Lowndes' Method, natural divisor = $118,284 \div 12 = 9857$

Store	Shoppers	Nat. Quota	In Alloc	Rel. Frac. Part	Fin Alloc
Boston	28,772	2.9189	2	$0.9189 \div 2 = 0.4595$ ✓	3
Mansfield	13,472	1.3667	1	$0.3667 \div 1 = 0.3667$ ✓	2
Providence	32,871	3.3348	3	$0.3348 \div 3 = 0.1116$	3
Hartford	43,169	4.3795	4	$0.3795 \div 4 = 0.0949$	4
<u>totals:</u>	<u>118,284</u>		<u>10</u>		<u>12</u>

(* final allocation based on largest rel. frac. part)

c.) Jefferson's Method, natural divisor = $118,284 \div 12 = 9857$

Store	Shoppers	Nat. Quota	In. Alloc	Thres. Div	Mod Quota	Fin Alloc
Boston	28,772	2.9189	2	$\frac{28,772}{3} = 9590.6667$ ①	3.3340	3
Mansfield	13,472	1.3667	1	$\frac{13,472}{2} = 6736.0000$ ②	1.5611	1
Providence	32,871	3.3348	3	$\frac{32,871}{4} = 8217.7500$ ③	3.8089	3
Hartford	43,169	4.3795	4	$\frac{43,169}{5} = 8633.8000$ ④	5.0022	5
<u>totals:</u>	<u>118,284</u>		<u>10</u>			<u>12</u>

Let md = 8630

3)
$$\frac{9 + (4.3)^{-2.9x}}{8} = 44$$

$$9 + (4.3)^{-2.9x} = 352$$

$$(4.3)^{-2.9x} = 343$$

$$-2.9x \log(4.3) = \log(343)$$

$$x = \frac{\log(343)}{-2.9 \log(4.3)} = -1.3801$$

4)
$$6(19-8y)^6 = 817$$

$$(19-8y)^6 = 136.1667$$

$$19-8y = (136.1667)^{1/6}$$

$$19-8y = 2.2682$$

$$-8y = -16.7318$$

✓ \$6500 @ 6.1% simple int

$$\begin{aligned}FV &= P(1+rt) \\&= 6500(1+(0.061)(10)) \\&= 6500(1+0.61) \\&= 6500(1.61) \\&= \$10,465\end{aligned}$$

\$6500 @ 5.6% compd monthly

$$\begin{aligned}FV &= P\left(1+\frac{r}{n}\right)^{nt} \\&= 6500\left(1+\frac{0.056}{12}\right)^{(12)(10)} \\&= 6500(1+0.004667)^{120} \\&= 6500(1.004667)^{120} \\&= 6500(1.7484) \\&= \$11,364.56\end{aligned}$$

so \$6500 @ 5.6% compd monthly is worth more than \$6500 @ 6.1% simple.

6) $FV = P\left(1+\frac{r}{n}\right)^{nt}$

$$2500 = 1500\left(1+\frac{0.0375}{12}\right)^{(12)t}$$

$$2500 = 1500(1+0.003125)^{12t}$$

$$2500 = 1500(1.003125)^{12t}$$

$$1.6667 = (1.003125)^{12t}$$

$$\log(1.6667) = 12t \log(1.003125)$$

$$t = \frac{\log(1.6667)}{12 \log(1.003125)} = 13.6433 \text{ yrs}$$

7) $FV = P\left(1+\frac{r}{n}\right)^{nt}$

$$1,000 = 42\left(1+\frac{r}{365}\right)^{(365)(6)}$$

$$1,000 = 42\left(1+\frac{r}{365}\right)^{2190}$$

$$13.8095 = \left(1+\frac{r}{365}\right)^{2190}$$

$$13.8095^{\frac{1}{2190}} = \left(1+\frac{r}{365}\right)$$

$$1.001449 = 1 + \frac{r}{365}$$

$$0.001449 = \frac{r}{365}$$

0.5287 = r so you would need an interest rate of 52.87%.

$$\begin{aligned}
 8) \quad FV &= D \left(\frac{(1 + \frac{r}{n})^{nt} - 1}{\frac{r}{n}} \right) \\
 &= 150 \left(\frac{(1 + \frac{0.045}{12})^{(12)(47)} - 1}{\frac{0.045}{12}} \right) \\
 &= 150 \left(\frac{(1 + 0.00375)^{564} - 1}{0.00375} \right) \\
 &= 150 \left(\frac{(1.00375)^{564} - 1}{0.00375} \right) \\
 &= 150 \left(\frac{8.2569 - 1}{0.00375} \right) \\
 &= 150 \left(\frac{7.2569}{0.00375} \right) \\
 &= 150 (1935.1624) \\
 &= \$ 290,274.36
 \end{aligned}$$

$$\begin{aligned}
 9) \quad FV &= D \left(\frac{(1 + \frac{r}{n})^{nt} - 1}{\frac{r}{n}} \right) \\
 50,000 &= D \left(\frac{(1 + \frac{0.045}{12})^{(12)(47)} - 1}{\frac{0.045}{12}} \right) \\
 50,000 &= D (1935.1624) \quad \leftarrow \text{from above in \# 8} \\
 \$ 387.56 &= D
 \end{aligned}$$

$$\begin{aligned}
 10) \quad P &= R \left(\frac{1 - (1 + \frac{r}{n})^{-nt}}{\frac{r}{n}} \right) \\
 300,000 &= R \left(\frac{1 - (1 + \frac{0.053}{12})^{-(12)(35)}}{\frac{0.053}{12}} \right) \\
 100,000 &= R \left(\frac{1 - (1 + 0.004417)^{-420}}{0.004417} \right) \\
 100,000 &= R \left(\frac{1 - (1.004417)^{-420}}{0.004417} \right) \\
 100,000 &= R \left(\frac{1 - 0.1571}{0.004417} \right) \\
 100,000 &= R \left(\frac{0.8429}{0.004417} \right) \\
 100,000 &= R (190.8324)
 \end{aligned}$$