Exam #2 Review

1. Solve for x:

$$2 + (9.2)^{-8x} = 2.32$$

Solution:

$$\begin{aligned}
2 + (9.2)^{-8x} &= 2.32 \\
(9.2)^{-8x} &= 2.32 - 2 \\
\log (9.2)^{-8x} &= \log .32 \\
-8x \log 9.2 &= \log .32 \\
\frac{-8x \log 9.2}{-8 \log 9.2} &= \frac{\log .32}{-8 \log 9.2} \\
x &= \boxed{.06418}
\end{aligned}$$

2. Solve for y:

$$3(y-7)^{13} = 1540$$

Solution:

$$\begin{aligned}
3(y-7)^{13} &= 1540 \\
(y-7)^{13} &= 513.33333 \\
y - 7 &= 1.61619 \\
y &= \boxed{8.61619}
\end{aligned}$$

3. Which will be worth more in 10 years: \$10,000 invested at 8.2% simple interest, or \$10,000 invested at 5% interest, compounded monthly?

Solution: For simple interest:

$$\begin{aligned} F &= (10000)(1 + (.082)(10)) \\ &= \boxed{\$18{,}200} \end{aligned}$$

For compound interest:

$$\begin{aligned} F &= (10000)\left(1 + \frac{.05}{12}\right)^{12\cdot 10} \\ &= (10000)(1.00416667)^{120} \\ &= (10000)(1.64701015) \\ &= \boxed{\$16{,}470.10} \end{aligned}$$

... so you earn more with simple interest.

4. Suppose a friend lends you \$100, and you agree to pay him back \$112 in 18 months. If we assume that this is <u>simple interest</u>, then what is the interest rate?

Solution: Note that 18 months is $t = 1.5$ years. Then solving for r,

$$\begin{aligned} F &= P(1 + rt) \\ 112 &= 100(1 + 1.5r) \\ 1.12 &= 1 + 1.5r \\ 0.12 &= 1.5r \\ 0.08 &= r = \boxed{8\%} \end{aligned}$$

5. For an account with an annual interest rate of 6%, find the annual percentage yield (APY) if interest is compounded:

   (a) quarterly?

   Solution:

$$\begin{aligned}\text{APY} &= \left(1 + \frac{.06}{4}\right)^4 - 1 \\ &= (1.015)^4 - 1 \\ &= 1.06136355 - 1 \\ &\approx .0614 = \boxed{6.14\%}\end{aligned}$$

   (b) monthly?

   Solution:

$$\begin{aligned}\text{APY} &= \left(1 + \frac{.06}{12}\right)^{12} - 1 \\ &= (1.005)^{12} - 1 \\ &= 1.06167781 - 1 \\ &\approx .0617 = \boxed{6.17\%}\end{aligned}$$

   (c) daily?

Solution:

$$\begin{aligned}\text{APY} &= \left(1 + \frac{.06}{365}\right)^{365} - 1 \\ &= (1.00016438)^{365} - 1 \\ &= 1.06182993 - 1 \\ &\approx .0618 = \boxed{6.18\%}\end{aligned}$$

6. A bank advertises a Certificate of Deposit (CD) with 4.8% interest, compounded monthly. If I invest \$3,500 today, how long will it take for my investment to grow to \$4,200?

Solution: Using the compound interest formula and solving for t,

$$\begin{aligned}
F &= P\left(1+\frac{r}{n}\right)^{nt} \\
4200 &= 3500\left(1+\frac{0.048}{12}\right)^{12t} \\
1.2 &= \left(1+\frac{0.048}{12}\right)^{12t} \\
\log(1.2) &= \log\left(\left(1+\frac{0.048}{12}\right)^{12t}\right) \\
\log(1.2) &= 12t\log\left(1+\frac{0.048}{12}\right) \\
\log(1.2) &= 12t\log(1.004) \\
\frac{\log(1.2)}{12\log(1.004)} &= t = \boxed{3.806 \text{ years}}
\end{aligned}$$

7. Reba would like to make the \$2,150 down payment on a new car in 6 months. If she has \$2,000 in her savings account, and interest is compounded daily, what interest rate would she need to earn to have enough?

Solution: Using the compound interest formula (t must be in years, not months):

$$\begin{aligned}
2150 &= (2000)\left(1+\frac{r}{365}\right)^{365\cdot\frac{6}{12}} \\
2150 &= (2000)\left(1+\frac{r}{365}\right)^{\frac{365}{2}} \\
1.075 &= \left(1+\frac{r}{365}\right)^{\frac{365}{2}} \\
(1.075)^{\frac{2}{365}} &= \left(\left(1+\frac{r}{365}\right)^{\frac{365}{2}}\right)^{\frac{2}{365}} \\
1.00039636 &= 1+\frac{r}{365} \\
0.00039636 &= \frac{r}{365} \\
0.00039636\cdot(365) &= r \\
0.14466994 &= r \approx \boxed{14.47\%}
\end{aligned}$$

8. When Jed was born, his grandfather deposited \$1,982 into a savings account for his grandson, under the condition that nobody touches it until Jed turns 21. If this account earns 3.9% interest compounded semi-anually (twice per year), then how much will Jed have on his 21st birthday?

Solution: Using the compound interest formula and solving for F,

$$\begin{aligned} F &= P\left(1+\frac{r}{n}\right)^{nt} \\ &= 1982\left(1+\frac{0.039}{2}\right)^{2\cdot 21} \\ &= 1982(1.0195)^{42} \\ &= 1982(2.25042) = \boxed{\$4{,}460.33} \end{aligned}$$

9. Many years later, Jed's granddaughter is born, and he would like to do something similar for her. He would like her to have exactly \$10,000 in the account on her 21st birthday. If the account earns 4.1% compounded annually, how much would Jed need to deposit on the day she is born?

Solution:

$$\begin{aligned} 10000 &= P\left(1+\frac{.041}{1}\right)^{1\cdot 21} \\ 10000 &= P(1.041)^{21} \\ 10000 &= P(2.32522680) \\ \frac{10000}{2.32522680} &= \frac{P(2.32522680)}{2.32522680} \\ P &= \boxed{\$4{,}300.66} \end{aligned}$$

10. It's never too early to start saving for retirement! Suppose you find a savings account that will pay 5% interest compounded monthly. If, starting on your next birthday, you deposit \$85 per month, and continue this until your 65th birthday, how much will you have in your account?

Solution: This depends on your current age, obviously, so let's assume you do this starting when you turn 21. Then on your 65th birthday you've been making deposits for $t = 65 - 21 = 44$ years. Using the systematic savings formula and solving for F,

$$\begin{aligned} F &= D\left(\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right) \\ &= 85\left(\frac{\left(1+\frac{0.05}{12}\right)^{12\cdot 44}-1}{\frac{0.05}{12}}\right) \\ &= 85\left(\frac{(1.0041667)^{528}-1}{0.0041667}\right) \\ &= 85\left(\frac{8.98386-1}{0.0041667}\right) \\ &= 85(1916.1258) = \boxed{\$162{,}870.69} \end{aligned}$$

11. Let's say you'd like to retire with, oh I don't know, \$1 million. Given the same account from #10, how much would you need to deposit every month for this to happen?

Solution: Assume you start on your 21st birthday.

$$\begin{aligned} 1000000 &= D\left(\frac{\left(1+\frac{.05}{12}\right)^{12\cdot 44}-1}{\frac{.05}{12}}\right) \\ 1000000 &= D\left(\frac{(1.00416667)^{528}-1}{(.00416667)}\right) \\ 1000000 &= D\left(\frac{7.98387327}{.00416667}\right) \\ 1000000 &= D(1916.128052) \\ \frac{1000000}{1916.128052} &= \frac{D(1916.128052)}{1916.128052} \\ D &= \boxed{\$521.89} \end{aligned}$$

12. Maggie borrows \$7,000 from the bank at 8% interest compounded monthly.

(a) If she makes a \$400 payment at the end of the first month, how much does she owe?

Solution: This is the "remaining balance" entry that would be at the end of the first row of an amortization schedule. It would read

Payment	Interest Paid	Principal Paid	Remaining Balance
\$400	$\frac{0.08}{12}(7000) = 46.67$	400-46.67 $=$ 353.33	7000-353.33 $=$ $\boxed{\$6{,}646.67}$

(b) If she continues paying \$400 monthly, how long will it take to pay off the loan?

Solution: Using the loan formula and solving for t,

$$\begin{aligned}
P &= R\left(\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right)\\
7000 &= 400\left(\frac{1-\left(1+\frac{0.08}{12}\right)^{-12t}}{\frac{0.08}{12}}\right)\\
17.5 &= \frac{1-(1.006667)^{-12t}}{0.006667}\\
0.116667 &= 1-(1.006667)^{-12t}\\
-0.883333 &= -(1.006667)^{-12t}\\
\log(0.883333) &= \log(\left((1.006667)^{-12t}\right)\\
\log(0.883333) &= -12t\log((1.006667)\\
\frac{\log(0.883333)}{-12\log(1.006667)} &= t = \boxed{1.556 \text{ years}}
\end{aligned}$$

13. Andrew takes out an \$18,500 student loan to pay for graduate school. If the interest rate is 6.3% compounded quarterly, how large would his quarterly payments be in order to pay off this loan in 10 years?

Solution: Using the loan formula and solving for R,

$$\begin{aligned}
P &= R\left(\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right) \\
18500 &= R\left(\frac{1-\left(1+\frac{.063}{4}\right)^{-4\cdot 10}}{\frac{.063}{4}}\right) \\
18500 &= R\left(\frac{1-(1.01575)^{-40}}{.01575}\right) \\
18500 &= R\left(\frac{.46478687}{.10575}\right) \\
R &= \frac{18500}{29.51027746} = \boxed{\$626.90}
\end{aligned}$$

14. Franny and Zooey are ready to buy their first house. They determine that they can pay \$1100 per month towards a mortgage. If the 20 year mortgage available to them charges 7.8% interest compounded monthly,

(a) how large of a loan can they afford?

Solution: Using the loan formula and solving for P,

$$\begin{aligned}
P &= R\left(\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right) \\
&= 1100\left(\frac{1-\left(1+\frac{0.078}{12}\right)^{-12\cdot 20}}{\frac{0.078}{12}}\right) \\
&= 1100\left(\frac{1-(1.0065)^{-240}}{0.0065}\right) \\
&= 1100\left(\frac{1-0.2111995}{0.0065}\right) \\
&= 1100\left(\frac{0.7888045}{0.0065}\right) = \boxed{\$133,489.31}
\end{aligned}$$

(b) create an amortization schedule for the first 3 months of the loan.

Solution: The rows detailing the first three monthly payments would read

Payment	Interest Paid	Principal Paid	Remaining Bal.
			\$133,489.31
1100	$\left(\frac{0.078}{12}\right)(133489.31) = \$867.68$	$1100 - 867.68 = \$232.32$	\$133,256.99
1100	$\left(\frac{0.078}{12}\right)(133256.99) = \$866.17$	$1100 - 867.68 = \$233.83$	\$133,023.16
1100	$\left(\frac{0.078}{12}\right)(133023.16) = \$864.65$	$1100 - 864.65 = \$235.35$	\$132,787.81

15. When rolling two dice, what is the probability that you:

(a) Roll a 5?

Solution: You could have (1,4), (2,3), (3,2), (4,1) as possible rolls. There are 36 total possibilities, so $P(5) = \frac{4}{36} = \frac{1}{9}$.

(b) Roll a number higher than 9?

Solution: You could have (4,6), (5,5), (6,4), (5,6), (6,5), (6,6) as possible roll, so $P(\geq 9) = \frac{6}{36} = \frac{1}{6}$.

(c) Don't roll a 3?

Solution: We know we could have (1,2) or (2,1) as our possible outcomes for rolling a 3, so to find $P(\text{not } 3)$, we have $P(\text{not } 3) = 1 - P(3) = 1 - \frac{2}{36} = \frac{34}{36} = \frac{17}{18}$.

(d) Roll a number that is at least 5?

Solution: We could consider all outcomes that result in 5 or higher, but there are a lot of those, so it will be faster to notice that the only outcomes which are *not* 5 or higher are the ones which are less than 5, i.e. (1,1), (1,2), (1,3), (2,1), (2,2), and (3,1). So $P(\geq 5) = 1 - P(\leq 4) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{15}{18}$.

(e) Roll an even number or a number larger than 3?

Solution: Use the addition rule. Clearly half of the possible outcomes are even and half are odd, so $P(\text{even}) = \frac{18}{36}$. $P(> 3) = \frac{33}{36}$, since every outcome except for (1,1), (2,1), and (1,2) is larger than 3. Of the 18 even outcomes, only (1,1) is not larger than 3, so $P(\text{even AND } > 3) = \frac{17}{36}$, and thus our final answer is $P(\text{even OR } > 3) = \frac{18}{36} + \frac{33}{36} - \frac{17}{36} = \frac{34}{36} = \frac{17}{18}$.

16. According to the American Medical Association, in 1996 there were 737,764 physicians in the United Sates, 157,387 of whom were female. There were 133,005 physicians under 35 years of age, 47,348 of whom were female. What is the probability that a randomly chosen physician in 1996 was female or under the age of 35?

Solution:

$$\begin{aligned} P(\text{female OR } \le 35) &= P(\text{female}) + P(\le 35) - P(\text{female AND } \le 35) \\ &= \frac{157387}{737764} + \frac{133005}{737764} - \frac{47348}{737764} \\ &= 0.21333 + 0.18028 - 0.064178 = 0.329432 = \boxed{32.9\%} \end{aligned}$$

17. A recent poll at a university shows that, in a vote for the new mascot: 60% of students would approve of a giraffe, 42% would approve of a hippo, and 17% would approve of both. If we select a student at random, what's the probability that he or she would approve of neither the giraffe nor the hippo?

Solution: Notice that $P(\text{neither A nor B}) = 1 - P(\text{either A or B})$... Then

$$P(G \text{ or } H) = P(G) + P(H) - P(G \text{ and } H) = 0.60 + 0.42 - 0.17 = 0.85,$$

so, $P(\text{neither } G \text{ nor } H) = 1 - 0.85 = 0.15 = 15\%$.