Work carefully, and **SHOW ALL STEPS** in order to receive full credit. An answer without supporting work will not receive credit. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name and section: _____

Instructor's name: _____

1. A PTA board voted to select how to spend $20,000 they had earned in a fund-raiser. The preference rankings of the voting members appear next.

	Number of Voters								
	1	**3**	**1**	**3**	**3**	**2**	**1**	**6**	**2**
Computers	1✓	1✓	2✓	2	3	3	3	2✓	3
Library Books	2	3	1✓	1✓	1✓	2	4	3	2
Playground Equipment	4	4	3	4	4	1✓	1✓	4	4
Science Lab	3	2✓	4	3	2✓	4	2✓	1✓	1✓

(a) Which choice would win using the plurality with runoff between the top two?

Solution:

Computers	$1 + 3 = 4$
Library Books	$1 + 3 + 3 = 7$
Playground Equipment	$2 + 1 = 3$
Science Lab	$6 + 2 = 8$

So **Science Lab** wins the plurality.

The **Runoff**:

Library Books	$7 + 1 + 2 = 10$
Science Lab	$8 + 3 + 1 = 12$

So **Science Lab** is the runoff winner.

(b) Which choice would win using Borda's method?

Solution:

Computers	$4 \cdot 1 + 4 \cdot 3 + 3 \cdot 1 + 3 \cdot 3 + 2 \cdot 3 + 2 \cdot 2 + 2 \cdot 1 + 3 \cdot 6 + 2 \cdot 2 = 62$
Library Books	$3 \cdot 1 + 2 \cdot 3 + 4 \cdot 1 + 4 \cdot 3 + 4 \cdot 3 + 3 \cdot 2 + 1 \cdot 1 + 2 \cdot 6 + 3 \cdot 2 = 62$
Playground Equipment	$1 \cdot 1 + 1 \cdot 3 + 2 \cdot 1 + 1 \cdot 3 + 1 \cdot 3 + 4 \cdot 2 + 4 \cdot 1 + 1 \cdot 6 + 1 \cdot 2 = 32$
Science Lab	$2 \cdot 1 + 3 \cdot 3 + 1 \cdot 1 + 2 \cdot 3 + 3 \cdot 3 + 1 \cdot 2 + 3 \cdot 1 + 4 \cdot 6 + 4 \cdot 2 = 64$

So **Science Lab** wins the Borda's count.

(c) If Borda's method is used, could the three members who ranked library books first, science lab second, computers third, and playground equipment last obtain a preferable result by voting strategically if the other members voted as shown in the table? Explain.

Explain your reasoning clearly. If your answer is "yes", indicate how the preferable outcome is obtained. If your answer is "no", explain why no preferable outcome is possible.

Solution: Yes. By switching their rankings for Computers, Playground Equipment, Science Lab so that they had the following column

4
1
2
3

This would result in the following Borda counts

Computers	59
Library Books	62
Playground Equipment	38
Science Lab	61

So **Library Books** would win, a preferable out come.

(d) Show all necessary Condorcet comparison's and then determine which choice, if any, would be a Condorcet winner?

Solution: From part (a) we have

Library Books	$7 + 1 + 2 =$	10
Science Lab	$8 + 3 + 1 =$	12

Computers	$4 + 1 + 3 + 2 =$	10
Science Lab	$8 + 3 + 1 =$	12

Playground Equipment	$3 + 1 =$	4
Science Lab	$8 + 1 + 3 + 3 + 3 =$	18

So **Science Lab** is the Condorcet winner.

(e) Which choice would win in an approval vote?

Solution:

Computers	$1 + 3 + 1 + 6 =$	11
Library Books	$1 + 3 + 3 =$	7
Playground Equipment	$2 + 1 =$	3
Science Lab	$3 + 3 + 1 + 6 + 2 =$	15

So **Science Lab** wins the approval vote.

(f) If an approval vote is used, could the 3 voters who ranked computer first and science lab second, library books third, and playground equipment fourth obtain a preferable result by voting strategically if the other members voted as shown in the table? Explain.

Explain your reasoning clearly. If your answer is "yes", indicate how the preferable outcome is obtained. If your answer is "no", explain why no preferable outcome is possible.

Solution: No. At best they could remove their support from Science Lab and the finally tally would be

Computers	$1 + 3 + 1 + 6 =$	11
Library Books	$1 + 3 + 3 =$	7
Playground Equipment	$2 + 1 =$	3
Science Lab	$3 + 1 + 6 + 2 =$	12

So **Science Lab** still wins.

2. Suppose that three candidate - Rosen, Brown, and Wheatley - are running in an election that will be decided by the plurality method with a runoff between the top two finishers if none of the candidates receives a majority of the votes. The results of the first ballot are given below.

Rosen	2246
Brown	5680
Wheatley	6126

In a runoff election between Wheatley and Brown, what percentage of Rosen supporters would need to vote for Brown in order for Brown to win the election? How about for Wheatley?

Solution: First we want to find the number of Rosen supporters Brown needs to win the election. In order for Brown to catch up to Wheatley Brown needs 446 votes from Rosen supporters. This leaves 1800 votes to be decided. To attain a majority a candidate would need $(1800/2)+1$ votes, so 901 votes. This means that Brown needs $446 + 901 = 1347$ vote. As a percentage of Rosen's supporters this is $1347/2246 = .5997$, so approximately 60%. Wheatley would only need the 901 votes to guarantee a majority. As a percentage of Rosen's supporters this is $901/2246 = .4029$, so approximately 40%.

3. Suppose there are 140 votes cast in an election among five candidates- Segarra, Perez, Peters, Perry, and Milner- to be decided by plurality. After the first 100 votes are counted, the tallies are as follows:

Segarra	12
Perez	23
Peters	17
Perry	29
Milner	19

(a) At this point, is it possible for anyone to win with a majority of the votes?

Solution: No. Even if Perry, the current front-runner, received the remaining 40 votes, his tally would be 69 total. With 140 votes, a majority winner would need to win at least 71 votes.

(b) What is the minimum number of remaining votes Perry needs to be assured of a win?

Solution: Let x = the number of additional votes for Perry. Then we need:

$$\begin{array}{rcl} \underline{\text{Perry}} & & \underline{\text{Perez}} \\ 29+x & > & 23+(40-x) \\ 2x & > & 34 \\ x & > & 17 \end{array}$$

We see that Perry needs *more than* 17 votes to ensure a win. Thus, 18 additional votes would be the minimum number.

(c) What is the minimum number of remaining votes Peters needs to be assured of a win?

Solution: Let x = the number of additional votes for Peters. Then we need:

$$\begin{array}{rcl} \underline{\text{Peters}} & & \underline{\text{Perry}} \\ 17+x & > & 29+(40-x) \\ 2x & > & 52 \\ x & > & 26 \end{array}$$

We see that Perry needs *more than* 26 votes to ensure a win. Thus, 27 additional votes would be the minimum number.

4. Based on the enrollment figures from the previous year, a junior college expects the total enrollment in mathematics courses, broken down by course, to be as given in the table.

   The college employs 11 mathematics professors and they each teach 4 classes, so a total of 44 classes will be offered. The mathematics department chair decides to apportion the 44 classes among the 5 courses according to the expected enrollment.

   (a) Apportion the classes using Hamilton's method.

Course	Total of Enrollment	Natural Quota D=	Initial Allocation	Final Allocation
Algebra	315			
Trigonometry	151			
Precalculus	698			
Calculus I	590			
Calculus II	305			
Total				

Solution:

Course	Total of Enrollment	Natural Quota D= 46.7955	Initial Allocation	Final Allocation
Algebra	315	6.7314	6	7
Trigonometry	151	3.2268	3	3
Precalculus	698	14.9160	14	15
Calculus I	590	12.6080	12	13
Calculus II	305	6.5177	6	6
Total	2059		41	44

   (b) Apportion the classes using Lowndes' method.

Course	Total of Enrollment	Natural Quota D=	Initial Allocation	Relative Fractional part	Final Allocation
Algebra	315				
Trigonometry	151				
Precalculus	698				
Calculus I	590				
Calculus II	305				
Total					

Solution:

Course	Total of Enrollment	Natural Quota D= 46.7955	Initial Allocation	Relative Fractional part	Final Allocation
Algebra	315	6.7314	6	0.12190	7
Trigonometry	151	3.2268	3	0.07560	4
Precalculus	698	14.9160	14	0.06543	14
Calculus I	590	12.6080	12	0.05067	12
Calculus II	305	6.5177	6	0.08628	7
Total	2059		41		44

5. Don't-Fall-or-you-Might-Get-Trampled-Mart has 11 truck-loads of flat-screen TV's to distribute to its 4 stores. A truck-load can't be split so they want to apportion these shipments based on the number of shoppers a store gets each week. These figures are given in the following table:

Store	Boston	Mansfield	Providence	Hartford
Shoppers per week	5430	2270	3600	4100

(a) Apportion these truckloads using Webster's method:

Store	**Shoppers**	**Natural Quota D=**	**Initial Allocation**	**Modified Quota D=**	**Final Allocation**
Boston	5430				
Mansfield	2270				
Providence	3600				
Hartford	4100				
Total	15400				11

Store	**Desired Allocation**	**Threshold Divisor**
Boston		
Mansfield		
Providence		
Hartford		

Solution:

Store	**Shoppers**	**Natural Quota D=1400**	**Initial Allocation**	**Modified Quota D=1450**	**Final Allocation**
Boston	5430	3.8786	4	3.7443	4
Mansfield	2270	1.6214	2	1.5655	2
Providence	3600	2.5714	3	2.4828	2
Hartford	4100	2.9286	2	2.8276	3
Total	15400				11

Store	**Desired Allocation**	**Threshold Divisor**
Boston	3.5	1551.4286
Mansfield	1.5	1513.3333
Providence	2.5	1440
Hartford	2.5	1640

(b) Apportion these truckloads using Jefferson's method:

Store	Shoppers	Natural Quota D=	Initial Allocation	Modified Quota D=	Final Allocation
Boston	5430				
Mansfield	2270				
Providence	3600				
Hartford	4100				
Total	15400				11

Store	Desired Allocation	Threshold Divisor
Boston		
Mansfield		
Providence		
Hartford		

Solution:

Store	Shoppers	Natural Quota D=1400	Initial Allocation	Modified Quota D=1150	Final Allocation
Boston	5430	3.8786	3	4.7217	4
Mansfield	2270	1.6214	1	1.9739	1
Providence	3600	2.5714	2	3.1304	3
Hartford	4100	2.9286	2	3.5652	3
Total	15400				11

Store	Desired Allocation	Threshold Divisor
Boston	4	1357.5
Mansfield	2	1135
Providence	3	1200
Hartford	3	1366.6667