
Beginning Probability

1. Consider rolling 2 six-sided dice and taking the sum of their values.
(*hint: There are 36 possible outcomes*)

What is the probability of:

- (a) obtaining a sum of 6?

Solution: There are 5 ways to roll a “6”, and 36 possible outcomes when rolling 2 dice. (See the table in the book)

$$P(\text{rolling a 6}) = \frac{5}{36}$$

- (b) getting a sum of 12?

Solution: $P(\text{rolling a 12}) = \frac{1}{36}$

- (c) rolling a value less than 5?

Solution: Rolling a 2,3, or 4 would satisfy this. Counting all of the ways to do this, we see:

$$P(\text{rolling less than 5}) = \frac{6}{36}$$

- (d) obtaining a value which is not 7?

Solution: We can use the “not” rule here:

$$P(\text{not rolling a 7}) = 1 - P(\text{rolling a 7}) = 1 - \frac{6}{36} = \frac{30}{36}$$

2. Consider drawing a single card from a standard deck. Decide whether each pair of events is mutually exclusive:

- (a) Event A = Drawing a King
Event B = Drawing a Queen

Solution: Since we can't draw one card that's *both* a King *and* a Queen, these events are mutually exclusive.

- (b) Event A = Drawing a Red Card
Event B = Drawing an Ace

Solution: On the other hand, there are cards which are both red and aces (namely the Ace of Diamonds and the Ace of Hearts), so these events are *not* mutually exclusive.

3. An iPod has 150 dance songs, 215 songs that are longer than four minutes, and 55 songs that are both dance songs and longer than for minutes. If this iPod has 1000 songs total and you set it on random, what is the probability that the next song will be a dance song *or* a song that is longer than four minutes?

Solution: We'll use the addition formula:

$$\begin{aligned}
 P(A \text{ or } B) &= P(A) + P(B) - P(\text{both A and B}) \\
 P(\text{Dance Song or Long Song}) &= P(\text{Dance Song}) + P(\text{Long Song}) - P(\text{Long Dance Song}) \\
 &= \frac{150}{1000} + \frac{215}{1000} - \frac{55}{1000} \\
 &= \frac{310}{1000} = 31\%
 \end{aligned}$$

4. On May 31, 1997, there were 487,297 personnel on active duty in the U.S. Army. Of these, 70,690 were women, 67,986 were commissioned officers, and 9,716 were women commissioned officers. What is the probability that a randomly selected member of the U.S. Army on this day was:
- (a) Neither a woman nor a commissioned officer?

Solution: If we think carefully about this, we'll see that the only type of soldier that satisfies these criteria is a male, non-officer. Given the information above, we don't directly have a way to count them.

Instead, we'll use that fact that (*neither A nor B*) is the complement of (*A or B*). i.e., in mathematical terms:

$$P(\text{neither A nor B}) = 1 - P(A \text{ or } B)$$

that is,

$$P(\text{neither Woman nor Officer}) = 1 - P(\text{Woman or Officer})$$

But we can find the probability on the right hand side by using the addition rule:

$$\begin{aligned}
 P(\text{Woman or Officer}) &= P(\text{Woman}) + P(\text{Officer}) - P(\text{Female Officer}) \\
 &= \frac{70690}{487297} + \frac{67986}{487297} - \frac{9716}{487297} \\
 &= \frac{128960}{487297}
 \end{aligned}$$

So

$$P(\text{neither Woman nor Officer}) = 1 - \frac{128960}{487297} = \frac{128960}{487297} \approx 73.5\%$$

(b) A man or a commissioned officer?

Solution: It's important to note how this question is different from the one in part (a). We'll use subtraction to find the numbers of male soldiers and male officers

$$\begin{aligned}P(\text{Man or Officer}) &= P(\text{Man}) + P(\text{Officer}) - P(\text{Male Officer}) \\&= \frac{416607}{487297} + \frac{67986}{487297} - \frac{58270}{487297} \\&= \frac{426323}{487297} \approx 87.5\%\end{aligned}$$

5. In a certain elementary school class, a survey is given and we find that: $\frac{1}{3}$ of the children like hot dogs, $\frac{1}{2}$ of the children like pizza, and $\frac{3}{4}$ of them like hot dogs *or* pizza. If a student is selected at random from this class, what is the probability that he or she likes both hot dogs *and* pizza?

Solution: Here, we'll use a bit of algebra to manipulate the Addition Rule formula:

$$\begin{aligned}P(A \text{ or } B) &= P(A) + P(B) - P(\text{both A and B}) \\P(\text{both A and B}) &= P(A) + P(B) - P(A \text{ or } B)\end{aligned}$$

So we can plug in our situation:

$$\begin{aligned}P(\text{both Hot Dogs and Pizza}) &= P(\text{Hot Dogs}) + P(\text{Pizza}) - P(\text{Hot Dogs or Pizza}) \\&= \frac{1}{3} + \frac{1}{2} - \frac{3}{4} \\&= \frac{1}{12}\end{aligned}$$

So there is a probability of $\frac{1}{12}$ (about 8.3%) That we choose a student who likes both hot dogs *and* pizza.