Exam 3 Review

1. A restaurant buys a box of 12 dozen eggs. Nine of the eggs were cracked in transit. If an egg is selected at random, what is the probability of choosing an un-cracked egg?

Solution: There are 9 cracked eggs out of 144 so the probability of selecting an un-cracked egg is $\frac{135}{144} = 93.7\%$

2. The Atlantis High School diving team has 24 divers. Of these, 18 are female and 9 are seniors, including 6 female seniors.

(a) What is the probability that a randomly selected diver is a female or a senior?

Solution:
P(Female or Senior) = P(Female) + P(Senior) - P(Both) $= \frac{18}{24} + \frac{9}{24} - \frac{6}{24} = \frac{21}{24}$

(b) What is the complement of the event described in part (a)?

Solution: The complement of getting either a senior or a female would be "getting neither a senior nor a female" i.e. getting a male who is a freshman, sophomore or junior.

3. In a DJ's record collection, 60% of the records are rock albums, and 40% of his albums are from the 70's. 20% of the albums are 70's rock albums. If one album is selected at random, what's the probability that is neither a rock album nor from the 70's?

Solution:

$$\begin{aligned} P(\text{neither 70's nor rock}) &= 1 - P(\text{either 70's or rock}) \\ &= 1 - (P(\text{70's}) + P(\text{rock}) - P(\text{70's and rock})) \\ &= 1 - (.4 + .6 - .2) \\ &= 20\% \end{aligned}$$

4. The probability that a student passes statistics is $\frac{4}{5}$ and the probability that a student passes philosophy is also $\frac{4}{5}$. If we assume no dependance between passing or failing either of these classes:

(a) What is the probability that a student passes both?

Solution:

$$P(\text{Passes Both}) = P(\text{Passes Stats. AND Passes Phil.}) = \frac{4}{5} \cdot \frac{4}{5} = \frac{16}{25}$$

(b) What is the probability that a student passes neither?

Solution:

$$P(\text{Passes neither}) = P(\text{fails Stats. AND fails Phil.}) = \frac{1}{5}\cdot\frac{1}{5} = \frac{1}{25}$$

(c) What is the probability that a student passes exactly one?

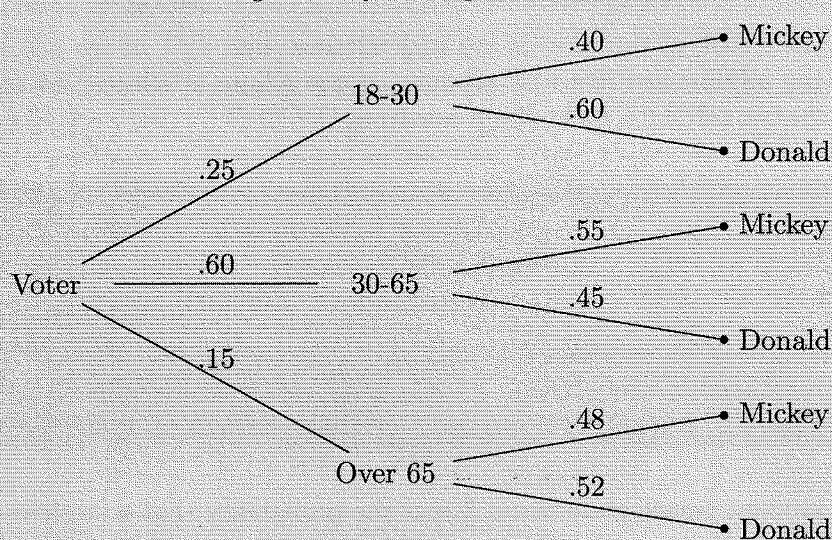
Solution:

$$\begin{aligned} P(\text{Passes one}) &= P((\text{Passes Stats. AND fails Phil.) OR (fails Stats. and Passes Phil.})) \\ &= \frac{4}{5}\cdot\frac{1}{5}+\frac{1}{5}\cdot\frac{4}{5} = \frac{8}{25} \end{aligned}$$

5. In a certain election, everyone voted for either Mickey Mouse or Donald Duck. The distribution of voters by age is:

Age	% of all voters	Voted for Mickey
18-30	25%	40 %
30 - 65	60%	55%
Over 65	15%	48%

Solution: A tree diagram may be helpful . . .



If a voter is selected at random, what is that probability that he or she:

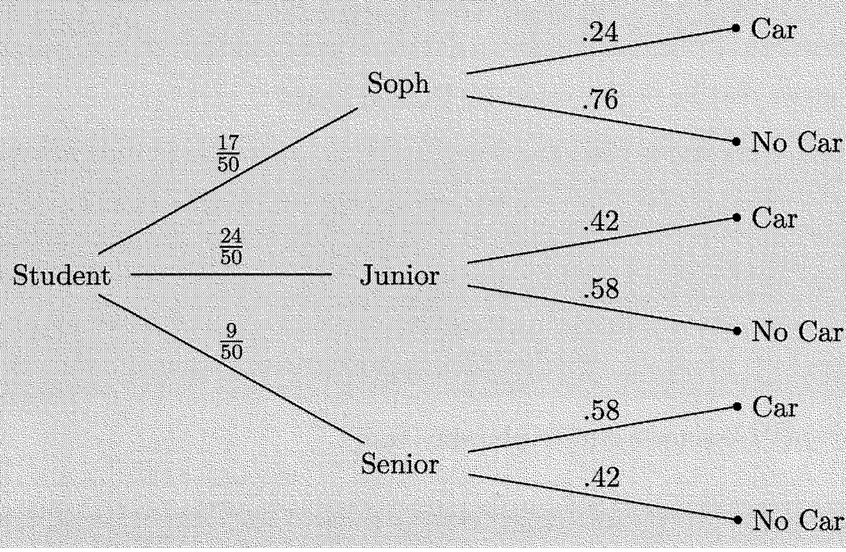
(a) is over 65 and voted for Donald Duck?

Solution:

$$P(\text{over 65 AND voted for DD}) = P(\text{over 65})\cdot P(\text{voted DD}|\text{over 65}) = (.15)(.52) = 7.8\%$$

6. At a certain college, 24% of sophomores have cars, 42% of juniors have cars, and 58% of seniors have cars. In a biology class at this school, there are 17 sophomores, 24 juniors, and 9 seniors. If a student is selected randomly from this class, that is the probability that he or she:

Solution: A tree diagram may be helpful . . .



(a) is a senior with a car?

Solution:

$$P(\text{Senior AND Car}) = P(\text{Senior}) \times P(\text{Car}|\text{Sen.}) = \left(\frac{9}{50}\right)(.58) = 10.44\%$$

(b) is a junior without a car?

Solution:

$$P(\text{Junior AND No Car}) = P(\text{Junior}) \times P(\text{No Car}|\text{Jun.}) = \left(\frac{24}{50}\right)(.58) = 27.84\%$$

(c) does not have a car?

Solution:

$$P(\text{No Car}) = \left(\frac{17}{50}\right)(.76) + \left(\frac{24}{50}\right)(.58) + \left(\frac{9}{50}\right)(.42) = 61.24\%$$

(d) is a sophomore given that he or she has a car?

(b) is between 30 - 65?

Solution: P(30-65) = 60%

(c) voted for Donald Duck?

Solution:

$$(.25)(.60) + (.60)(.45) + (.15)(.52) = 49.8\%$$

(d) is between 18-30, given that he or she voted for Donald Duck?

Solution: We need to use Bayes' theorem here:

$$\begin{aligned} P(\text{18-30}|\text{Donald}) &= \frac{P(\text{Donald}|\text{18-30}) \cdot P(\text{18-30})}{P(\text{Donald})} \\ &= \frac{(.60)\,(.25)}{.498} = 30.12\% \end{aligned}$$

Also, who won the election? (Assume a simple plurality vote).

Solution: If donald got 49.8% of the vote, then Mickey got the other 50.2%. It was close, but Mickey won.

Solution: We need to use Bayes' theorem here:

$$\begin{aligned} P(\text{Soph}|\text{Car}) &= \frac{P(\text{Car}|\text{Soph}) \cdot P(\text{Soph})}{P(\text{Car})} \\ &= \frac{(.24)\left(\frac{17}{50}\right)}{.3876} = 21.05\% \end{aligned}$$

(e) is a senior given that he or she does not have a car?

Solution: Again, Bayes' Theorem:

$$\begin{aligned} P(\text{Senior}|\text{No Car}) &= \frac{P(\text{No Car}|\text{Sen}) \cdot P(\text{Sen.})}{P(\text{No Car})} \\ &= \frac{(.42)\left(\frac{9}{50}\right)}{.6124} = 12.34\% \end{aligned}$$

7. Evaluate (using the formulas and showing all steps):

(a) ${}_{10}P_7$

Solution:

$$\frac{10!}{(10-7)!} = \frac{10!}{3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 604800$$

(b) ${}_{11}C_3$

Solution:

$$\frac{11!}{3!(11-3)!} = \frac{11!}{3!8!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 11 \cdot 5 \cdot 3 = 165$$

8. A math question has eight questions. The teacher would like to give everyone the same 8 questions, but vary the order on different tests. In how many ways can this be done?

Solution: This is a strictly ordering question. The number of ways to order the 8 problems on the test is $8! = 40320$

9. A fast food restaurant offers 6 beverage choices, 5 different sandwiches, and 8 side dishes. In how many ways could a family order 3 different drinks, 3 different sandwiches, and 3 different sides?

Solution: We can apply the fundamental law of counting: There are $({}_6C_3) \cdot ({}_5C_3) \cdot ({}_8C_3)$ ways this could be done.

10. Consider the Powerball lottery: You choose 5 numbers from 1 - 59 (no repetitions), and then a separate "powerball" number from 1-39.

   (a) In how many ways can this be done?

   Solution:
   $$({}_{59}C_5)({}_{39}C_1) = ({}_{59}C_5) \cdot 39 = 195,249,054$$

   (b) If I play a ticket, what's the probability that I won't match any of the numbers drawn?

   Solution:
   $$\frac{({}_{54}C_5)({}_{38}C_1)}{({}_{59}C_5)({}_{39}C_1)} \approx 61.55\%$$

   (c) If I play a ticket, what's the probability that I will match exactly 3 of the regular numbers, and not match the powerball?

   Solution:
   $$\frac{({}_{5}C_3)({}_{54}C_2)({}_{38}C_1)}{({}_{59}C_5)({}_{39}C_1)} \approx 0.28\%$$

11. From a bag containing 7 red, 4 white, and 6 orange balls, we draw 3 at random. What is the probability that

    (a) we get 3 red balls?

    Solution:
    $$\frac{({}_7C_3)}{({}_{17}C_3)}$$

    (b) at least two of the three are of the same color?

    Solution: We could painstakingly examine every case included here, which would take a while. OR . . . we can make a clever observation:
    $$\begin{aligned} P(\text{at least 2 of the same}) &= 1 - P(\text{all three different}) \\ &= 1 - P(\text{ 1 red and 1 white and 1 orange}) \\ &= 1 - \frac{({}_7C_1)({}_4C_1)({}_6C_1)}{({}_{17}C_3)} \\ &= 1 - \frac{7 \cdot 4 \cdot 6}{680} = 24.71\% \end{aligned}$$

12. A bowl of candy contains only 8 cherry candies and 17 peppermint candies. If we select 5 pieces of candy at random, what is the probability that:

    (a) None of the candies are cherry?

    Solution:
    $$\frac{({}_{17}C_5)}{({}_{25}C_5)} \approx 11.65\%$$

    (b) Exactly one of the candies is cherry?

    Solution:
    $$\frac{({}_8C_1)({}_{17}C_4)}{({}_{25}C_5)} \approx 35.8\%$$

    (c) At least one of the candies is cherry?

    Solution:
    $$P(\text{at least one cherry}) = 1 - P(\text{no cherry}) = 1 - \frac{({}_{17}C_5)}{({}_{25}C_5)} \approx 88.35\%$$

    (d) All of the candies are cherry?

    Solution:
    $$P(\text{all 5 cherry}) = \frac{({}_8C_5)}{({}_{25}C_5)} \approx .001\%$$

13. The rules for the "Win or Lose" lottery are as follows: You pay \$2 to play, and choose a 3-digit number from 000 to 999 (repetitions are allowed). That evening, a 3-digit number is drawn on TV. You can win in 2 ways: You win \$1300 if your number matches the winning number exactly. You win \$1 if none of your numbers line up with the corresponding digits in the winning number. (For example, if the winning number was 423, you would lose if you played 521, but win \$1 if you had played 251.) So

    What's the expected value of playing this game?

    Solution: Notice that there are $10 \cdot 10 \cdot 10 = 1000$ possible numbers to play in this game. First, we'll examine the outcome of "winning big." This event carries a value of \$1300. There's only one way to match the winning number, so the probability of winning big is $\frac{1}{1000}$.

    Next look at "winning small." This event caries a value of just \$1. There are 9 ways to *not* match the first digit, 9 ways to *not* match the second, etc. Which means that there are $9 \cdot 9 \cdot 9 = 729$ ways to "win small" and the probability of doing so is $\frac{729}{1000}$.

    $$\text{Expected Winnings} = \left(\frac{1}{1000}\right) \times (\$1300) + \left(\frac{729}{1000}\right) \times (\$1) = \$2.029$$

    So after buying a \$2 ticket, the expected value is \$2.029 - \$2 = 2.9¢

14. a 55 year old male would like to buy a one-year, \$1,000,000 life insurance policy. The company's actuaries find that there is a probability of 0.00152 that he will die during this year. What should the company charge him for this policy if they want to make an expected profit of \$75?

    Solution: Their expected payment is:

    $$(\$1000000) \times (0.00152) = \$1520$$

    In order to cover this, and make a \$75 profit, they should charge him \$1520 + \$75 = \$1595.

15. Vitamin D-resistant Rickets is a sex-linked genetic disease caused by a dominant gene on the X chromosome. Let X_R denote an X chromosome carrying this disease, while X and Y will represent healthy chromosomes. (So women with gene type X_RX or X_RX_R and men with gene type X_RY will have the disease).

    Suppose a woman does not have vitamin D-resistant Rickets, but that her husband does have the disease.

    (a) Construct the corresponding Punnett Square.

    Solution:

	X_R	Y
X	X_RX	XY
X	X_RX	XY

    (b) If they are expecting a child, what is the probability that the baby will not have vitamin-D resistant Rickets?

Solution:

$$P(\text{healthy}) = \frac{1}{2}$$

(c) They know, through prenatal testing, that the child is a girl. What is the probability that she will have the disease?

Solution:

$$P(\text{Rickets} \mid \text{girl}) = 100\%$$