CHAPTER 1 - VOTING METHODS

1.3 HEAD-TO-HEAD COMPARISONS

Head-to-Head Comparison A *head-to-head comparison* is a plurality election between any two of the candidates.

Example 1: Look again at the example from 1.2, in which a basketball team votes on a team captain.

	Number of Voters					
	3	**1**	**1**	**1**	**2**	**1**
Thomas	1	1	2	3	2	3
Walker	2	3	1	1	3	2
Goodman	3	2	3	2	1	1

(1) Who would win in a head-to-head comparison between Thomas and Walker?

(2) Who would win in a head-to-head comparison between Walker and Goodman?

(3) Who would win in a head-to-head comparison between Thomas and Goodman?

How to solve this: In a head-to-head comparison problem, like in a runoff, you add up the two candidates first place votes, then look at the voters who selected candidates not in the race, and use their preference ranking to see how they would vote now.

(1) Thomas has 4 first place votes, and Walker has 2. The voters who picked Goodman as their first place choice now have to pick Thomas or Walker. The 2 in the second to last column picked Thomas as a second choice, so they would now pick Thomas. The 1 in the last column picked Walker as a second choice, so they would now pick Goodman. This gives Thomas$4+2=6$ votes, and Walker $2+1=3$ votes. Thomas would win.

(2) Walker has 2 first place votes, and Goodman has 3 first place votes. The first 3 voters for Thomas picked Walker as their second choice, so Walker gets $2+3=5$ votes. The voter in the second column picked Goodman second, so Goodman gets $3+1=4$ votes. Walker would win.

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(3) Thomas has 4 first place votes, and Goodman has 3. One of Walker's voters picked Thomas as a second choice, and the other picked Goodman, and so Thomas would get 5 votes and Goodman would get 4. Thomas would win.

Condorcet winner A candidate who is the winner of a head-to-head comparison with every other candidate is called a *Condorcet winner.* If a candidate beats or at least ties every other candidate in election, that candidate is a *weak Condorcet winner.* **Note:** An election may not have a Condorcet winner.

Example 2: In the 1990 Louisiana gubernatorial election, the three main candidates were Duke, a former KKK grand wizard, Edwards, an ex-governor twice indicted for fraud, and Roemer who had recently switched parties. In the election, Duke won 32% of the votes, Edwards won 34%, and Roemer won 27%. In a runoff election between Edwards and Duke, Edwards won. The following is an estimate of the voters' preference rankings (see page 36 in book to see how the estimates were made).

	Percentage of Voters					
	9	**28**	**11**	**23**	**23**	**6**
Edwards	1	1	2	3	2	3
Duke	2	3	1	1	3	2
Roemer	3	2	3	2	1	1

Who is the Condorcet winner of this election?

How to solve this: We have to check head-to-head comparisons between each of the candidates. First we can check Edwards and Duke. Edwards has 37% of the first place votes and Duke has 34%. The 23% of voters who picked Roemer first would pick Edwards as a second choice, and the 6% would pick Roemer, so Edwards would get 37%+23% = 60% and Duke would get 40%. Edwards would win over Duke.

Now look at Duke and Roemer. Duke has 34% of the first place votes and Roemer has 29%. Only 9% of voters pick Edwards first and Duke 2nd, while 28% picked Edwards first but would switch to Roemer. Thus Duke would get 34% + 9% = 43% and Roemer would get 57%. Roemer would win over Duke.

Between Edwards and Roemer, Edwards has 37% of the first place votes and Roemer has 29%. The 11% of voters who picked Duke first and Edwards 2nd would pick Edwards now, and 23% would pick Roemer now. So Edwards would get 37% + 11% = 48% of the votes and Roemer would get 52% of the votes. Roemer would win over Edwards.

Roemer won in each head-to-head comparison, so he is the Condorcet winner.]

Note: In a problem with more candidates, you may not want to check all the head-to-head comparisons, because there may be a lot. You may want to be clever about which comparisons you check. A good hint is that as soon as a candidate loses one head-to-head comparison, they can't be the Condorcet winner, and so you can cross them off as a possibility. I suggest starting with the top candidates from a plurality election and checking them against each other. Whichever loses is ruled out, so you can start checking the winner against other candidates.

Example 3: (No Condorcet Winner)
Suppose the following are the preference rankings of voters in a vote on the best flavor of ice cream.

	Number of Voters		
	3	**2**	**2**
Chocolate	1	3	2
Vanilla	2	1	3
Strawberry	3	2	1

In a head-to-head comparison between chocolate and vanilla, chocolate wins with 5 votes. In a head-to-head comparison between vanilla and strawberry, vanilla wins with 5 votes. In a head-to-head comparison between chocolate and strawberry, strawberry wins with 4 votes. There is no Condorcet winner.

Definitions:

Head-to-head comparison:

Condorcet Winner:

Examples:

Head-to-Head Comparisons: In an example from section 1.2 we saw a five member committee trying to select a chair from among three candidates. We were given the following preference rankings

	1	**2**	**1**	**1**
Coleman	1	2	2	3
Horowitz	2	1	3	2
Taylor	3	3	1	1

Is there a Condorcet winner? In other words:

1. Who would be the winner in a head-to-head comparison between Coleman and Horowits?
2. Who would be the winner in a head-to-head comparison between Coleman and Taylor?
3. Who would be the winner in a head-to-head comparison between Horowitz and Taylor?
4. Finally is there a candidate that wins in all head-to-head comparisons they are included in, i.e. is there a Condorcet winner?

No Condorcet Winner: The managers of a catering service meet to decide on their first ever employee-of-the-month. Three employees are suggested: Julia, Paul, and Wolfgang. The managers' preference rankings are:

	1	**1**	**3**	**2**
Julia	1	1	3	2
Paul	2	3	1	3
Wolfgang	3	2	2	1

Which employee, if any, is the Condorcet winner?

Comparing Voting Methods: The parents of children in an elementary school were asked to vote on having mandatory school uniforms, optional school uniforms, or no school uniforms. The preference rankings of the parents are as follows:

	36	**5**	**16**	**19**	**1**	**42**
Mandatory uniforms	1	1	2	3	2	3
Optional uniforms	2	3	1	1	3	2
No uniforms	3	2	3	2	1	1

1. Which option has the top Borda count?
2. Which option would win a plurality of the vote?
3. Which option would win using a plurality with runoff between the top two finishers?
4. Which option, if any, is the Condorcet winner?