Plurality and Run-off Methods

1. Twelve friends would like to see a movie, so they vote on their options. Their preferences are as follows:

	Number of Voters						
	1	**1**	**1**	**1**	**3**	**3**	**2**
Don't be Afraid of the Dark	1	1	2	3	2	2	3
Rise of the Planet of the Apes	2	4	1	2	3	4	2
Our Idiot Brother	3	2	3	1	1	3	4
30 Minutes or Less	4	3	4	4	4	1	1

   (a) If they use a simple plurality vote, which movie would win?

Solution:

Afraid of the Dark	2
Apes	1
Idiot Brother	4
$\boxed{\text{30 Minutes}}$	5

   (b) Assuming that everyone else votes the same way, could the two people who ranked "Afraid of the Dark" as # 1 have voted strategically to achieve a more preferable outcome?

Solution: If they both switched their votes to "Our Idiot Brother," this movie would win, and each of them would prefer this over "30 Minutes."

   (c) Suppose that "30 Minutes or Less" is sold out. Which of the remaining three choices would win a simple plurality vote?

Solution:

$\boxed{\text{Afraid of the Dark}}$	2+3
Apes	1+2
Idiot Brother	4

   (d) If they do a plurality vote with a run-off between the top 2, which movie would they see? (Assume all movies are available).

Solution:

$\boxed{\text{Idiot Brother}}$	1+1+1+1+3 = 7
30 Minutes	2+2+1=5

2. Suppose that 100 votes are cast in an election between three candidates- Coltrane, Davis, Evans- to be decided by plurality. The first 70 votes are counted as follows:

Coltrane	39
Davis	23
Evans	8

(a) What is the minimum number of the remaining votes that Coltrane needs to ensure a win?

Solution:

$$\begin{aligned} 39 + x &> 23 + (30 - x) \\ 2x &> 14 \\ x &> 7 \end{aligned}$$

He needs at least 8 votes.

(b) What is the minimum number of the remaining votes that Davis needs to ensure a win?

Solution:

$$\begin{aligned} 23 + x &> 39 + (30 - x) \\ 2x &> 46 \\ x &> 23 \end{aligned}$$

He needs at least 24 votes.

(c) What is the minimum number of the remaining votes that Evans needs to ensure a win?

Solution: Evans couldn't win, even if he got all of the remaining votes.

3. The class of 2010 at G. Clinton High School voted on their graduation song. The tally came in:

	Percentage of Class				
	18	**13**	**22**	**18**	**29**
Freebird	1	1	2	3	5
School's Out	2	3	1	2	3
I've Had the Time of my Life	5	5	5	4	2
Keep on Rockin' in the Free World	3	2	4	1	4
Theme from "Friends"	4	4	3	5	1

(a) Suppose they hold a plurality vote with run-offs among the top 3 songs. Does this yield a majority decision? How should they proceed?

Solution:

Freebird	31%	
School's Out	22%	
I've Had the Time of my Life	0%	No majority.
Keep on Rockin' in the Free World	18%	
Theme from "Friends"	29%	

(b) Now consider a plurality vote with a run-off between the top two:

i. Which song wins?

Solution:

Freebird	31% + 22% + 18% = 71%
Theme from "Friends"	29%

ii. Assuming all other votes stay the same, could the 22% of the class who chose "School's Out" vote strategically to achieve a preferable outcome?

Solution: No, they already got their 2nd choice, and they can't do anything more to help their first choice.

iii. Assuming all other votes stay the same, could the 18% of the class who chose "Rockin' in the Free World" vote strategically to achieve a preferable outcome?

Solution: Yes. If they voted for "School's Out" then this song would win both the plurality vote and the run-off. In this case they would get their 2nd choice instead of their 3rd.

iv. Assuming all other votes stay the same, could the 29% of the class who chose the theme from "Friends" vote strategically to achieve a preferable outcome?

Solution: They also could change their vote to "School's Out," which would then win, giving them their 3rd choice instead of their 5th.