Math 5511 Homework 2

Due May 10, 2019

Solve 2 problems:

(1) Given any $\alpha > 0$, a matrix A is defined by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha \\ 0 & 0 & \alpha \end{bmatrix}.$$

Let $\mathrm{lub}_2(A)$ be the matrix norm induced by the Euclidean vector norm and define the condition number by $\mathrm{cond}_2(A) = \mathrm{lub}_2(A^{-1})\mathrm{lub}_2(A)$. Prove that

$$\mathrm{cond}_2(A) = \frac{1 + 2\alpha^2 + \sqrt{1 + 4\alpha^4}}{2\alpha}.$$

(2) Let H be a nonsingular, $n \times n$ complex matrix in upper-Hessenberg form, with eigenvalues λ_i, $i = 1, 2, \ldots, n$. If $H = [h_{ij}]$, assume that some but not all entries $h_{i+1,i}$ are zero, $i = 1, 2, \ldots n-1$. Also, assume the eigenvalues are distinct. Explain precisely how to adapt Hyman's method to find the eigenvalues and eigenvectors of H. Hint: H is block-upper-triangular.

(3) Let A be a nonsingular, $n \times n$ complex matrix. You are given that

$$A = Q_1 R_1 = Q_2 R_2$$

are two QR-factorizations of A, with $Q_1^H Q_1 = Q_2^H Q_2 = I$ and both R_1 and R_2 are upper-triangular. Prove that there exists a phase matrix $S = \mathrm{diag}(e^{i\phi_1}, \ldots, e^{i\phi_n})$, $\phi_j \in \mathbb{R}$, $j = 1, 2, \ldots, n$ such that

$$Q_2 = Q_1 S^H, \quad R_2 = S R_1.$$

(4) Show that if the QR algorithm is performed with Givens rotations, the structures of Hessenberg and Hermitian tridiagonal matrices are preserved upon each iteration.