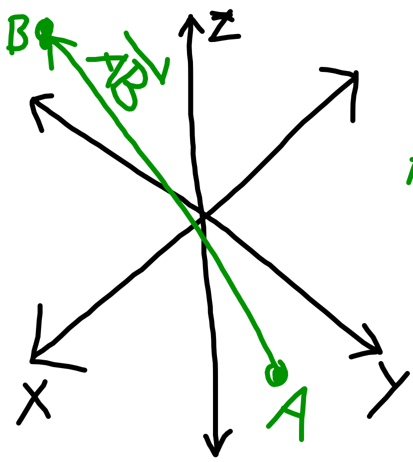


$\overrightarrow{AB}$ points from A to B

$A = (a_1, a_2, a_3)$

$B = (b_1, b_2, b_3)$

$$\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle.$$

Ex. 1: Let $A=(0,2,-1)$ and $B=(3,4,-6)$.

Find $\overrightarrow{AB}$.

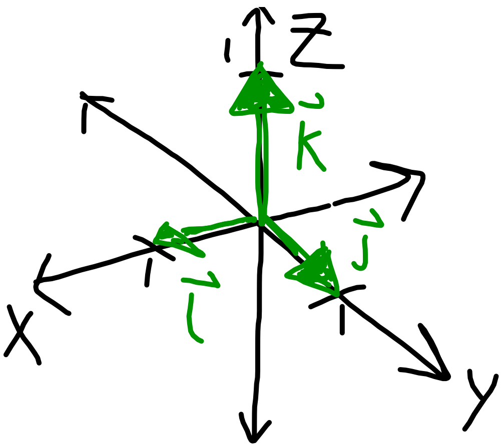
$$\overrightarrow{AB} = \langle 3-0, 4-2, -6-(-1) \rangle$$
$$= \langle 3, 2, -5 \rangle.$$

Standard basis vectors

$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$



Note that we may write

$$\langle a, b, c \rangle = a\langle 1,0,0\rangle + b\langle 0,1,0\rangle + c\langle 0,0,1\rangle$$
$$= a\vec{i} + b\vec{j} + c\vec{k}.$$

For example, $\langle -5, 6, 4 \rangle = -5\vec{i} + 6\vec{j} + 4\vec{k}$.

Normalizing vectors $\hat{v} = \underbrace{\frac{1}{|\vec{v}|}\vec{v}}$. } "unit vector"

$$|\hat{v}| = \left|\frac{1}{|\vec{v}|}\vec{v}\right| = \frac{1}{|\vec{v}|}|\vec{v}| = 1.$$

Ex. 2: Find a vector of length 2 in the direction of < 1, 1, -1 >. Let $\vec{v} = \langle 1, 1, -1\rangle$; we need $2\hat{v}$...

$$\frac{2}{|\vec{v}|}\vec{v} = \frac{2}{\sqrt{3}}\langle 1, 1, -1\rangle.$$

Ex. 3: Find $\hat{v}$ if $\vec{v} = \vec{j} - 2\vec{k}$.

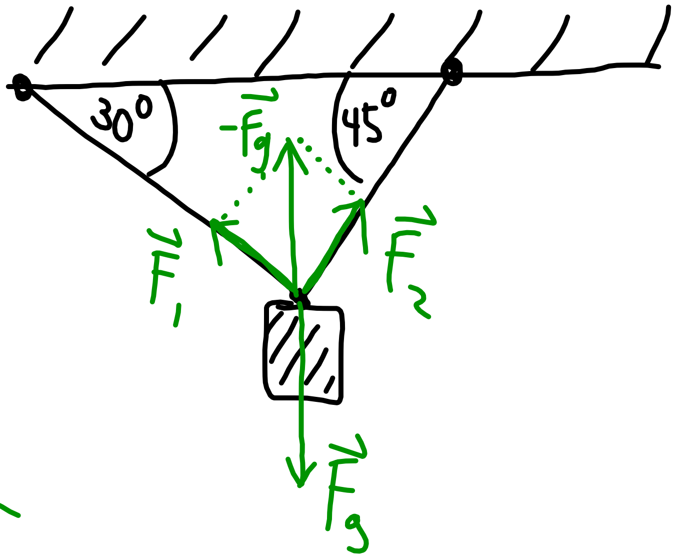
$$|\vec{v}| = \sqrt{0^2+1^2+2^2} = \sqrt{5}$$

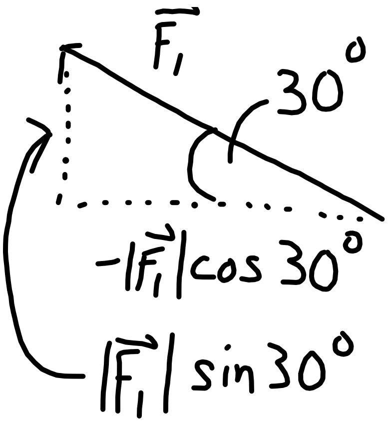
$$\Rightarrow \hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{5}}\langle 0, 1, -2\rangle$$

$$\text{or } \frac{1}{\sqrt{5}}\vec{j} - \frac{2}{\sqrt{5}}\vec{k}.$$

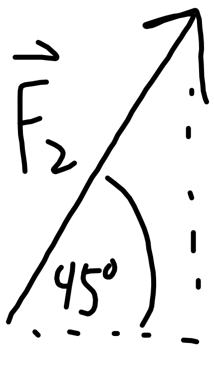
APPLICATION: If something is at rest, the net force is zero...

$$\vec{F}_1 + \vec{F}_2 = -\vec{F}_g$$





$$\vec{F}_1 = \left(-|\vec{F}_1|\cos 30^\circ\right)\vec{i} + \left(|\vec{F}_1|\sin 30^\circ\right)\vec{j}$$



$$\vec{F}_2 = |\vec{F}_2|\cos 45^\circ\,\vec{i} + |\vec{F}_2|\sin 45^\circ\,\vec{j}.$$

Recall $\vec{F}_1 + \vec{F}_2 = -\vec{F}_g = 20\vec{j}$ (example)

Trig... $\vec{F}_1 = -|\vec{F}_1|\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}|\vec{F}_1|\vec{j}$

$\vec{F}_2 = \frac{1}{\sqrt{2}}|\vec{F}_2|\vec{i} + \frac{1}{\sqrt{2}}|\vec{F}_2|\vec{j}$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}|\vec{F}_2| - |\vec{F}_1|\frac{\sqrt{3}}{2}\right)\vec{i} + \left(\frac{1}{\sqrt{2}}|\vec{F}_2| + \frac{1}{2}|\vec{F}_1|\right)\vec{j}$$

$$= 20\vec{j} + 0\vec{i}$$

$0 = \frac{1}{\sqrt{2}}|\vec{F}_2| - |\vec{F}_1|\frac{\sqrt{3}}{2}$ ... $20 = \frac{1}{\sqrt{2}}|\vec{F}_2| + \frac{1}{2}|\vec{F}_1|$

$$|\vec{F_1}|\frac{1}{2}+\frac{1}{\sqrt{2}}|\vec{F_2}|=20$$

$$-\frac{\sqrt{3}}{2}|\vec{F_1}|+\frac{1}{\sqrt{2}}|\vec{F_2}|=0 \Rightarrow |\vec{F_2}|=\frac{\sqrt{6}}{2}|\vec{F_1}|$$

$$\Rightarrow \frac{1}{2}|\vec{F_1}|+\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{6}}{2}|\vec{F_1}|=\frac{1}{2}(1+\sqrt{3})|\vec{F_1}|=20$$

$$\Rightarrow |\vec{F_1}|=\frac{40}{1+\sqrt{3}}$$

(Insert above to get $|\vec{F_2}|$).

DOT PRODUCTS $\vec{u}\vec{v}$?? (meaningless)

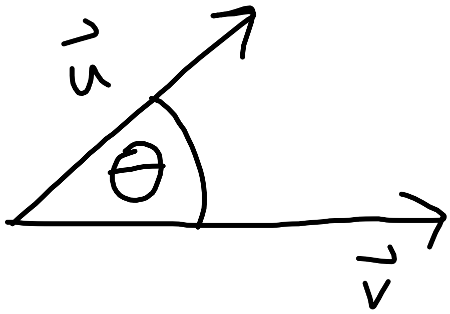
DOT: $\boxed{\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3}$

PROPERTIES: 1. $|\vec{u}|^2 = \vec{u}\cdot\vec{u}$

2. $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$

3. $\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$

4. $(c\vec{u})\cdot\vec{v} = c(\vec{u}\cdot\vec{v}) = \vec{u}\cdot(c\vec{v})$



$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right).$$

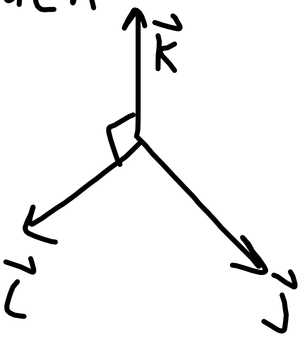
Ex. 4: Find the angle θ between $\vec{u} = \langle 1,1,1\rangle$ and $\vec{v} = \langle 2,0,1\rangle$. $\vec{u}\cdot\vec{v} = 1\cdot 2 + 1\cdot 0 + 1\cdot 1 = 3$

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{3}\cdot\sqrt{5}}\right) = \cos^{-1}\left(\sqrt{\frac{3}{5}}\right).$$

Orthogonal vectors lie at right angles to each other. "$\vec{u} \perp \vec{v}$"

"$\vec{u}$ is orthogonal to $\vec{v}$"

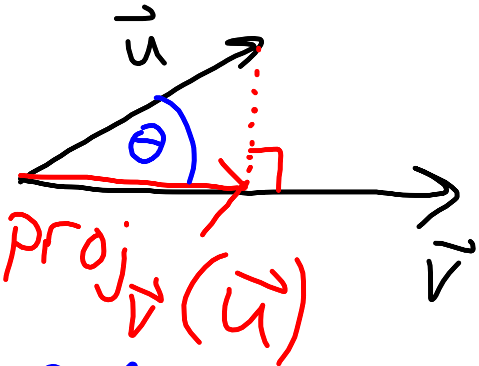
EX:



$$\cos 90^\circ = 0 = \vec{u} \cdot \vec{v}$$

EX: $\vec{i} \cdot \vec{j} = \langle 1,0,0 \rangle \cdot \langle 0,1,0 \rangle = 0$

Projections



$$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\,\hat{v}$$

$$= |\vec{u}|\,\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\,\hat{v} = \underbrace{\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}}_{\text{comp}_{\vec{v}}(\vec{u})}\,\hat{v}$$

$$\boxed{= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\,\vec{v}}$$

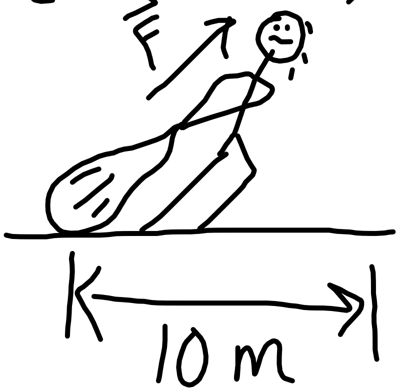
Ex. 5: $\vec{u} = \langle 2, 3, -4 \rangle$, $\vec{v} = \langle 0, 1, -1 \rangle$...

Find $\text{proj}_{\vec{v}} \vec{u}$.

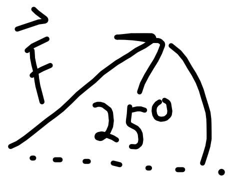
$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{3+4}{1+1} \langle 0, 1, -1 \rangle$$

$$= \left\langle 0, \frac{7}{2}, \frac{-7}{2} \right\rangle.$$

Calculation of the work done by a constant force.



$|\vec{F}| = 90N$



$$\vec{F} = 90\cos 25^\circ\, \vec{i} + 90\sin 25^\circ\, \vec{j}$$

$$\vec{D} = 10\vec{i}$$

$$W = \vec{F}\cdot\vec{D} = 900\cos 25^\circ$$

#1 Let $\vec{u} = \vec{i} + 2\vec{j} - 4\vec{k}$.

(a) Find $\hat{u}$. $|\vec{u}| = \sqrt{21}$

$$\hat{u} = \frac{1}{\sqrt{21}}\vec{u} = \frac{1}{\sqrt{21}}\vec{i} + \frac{2}{\sqrt{21}}\vec{j} - \frac{4}{\sqrt{21}}\vec{k}$$

(b) Find a vector of length 5 in the opposite direction of $\vec{u}$.

$$-5\hat{u} = \frac{-5}{\sqrt{21}}\vec{i} - \frac{10}{\sqrt{21}}\vec{j} + \frac{20}{\sqrt{21}}\vec{k}.$$

#2 Find $\overrightarrow{AB}$; $A=(-2,3,5)$, $B=(6,1,0)$.

$$\overrightarrow{AB} = \langle 6-(-2), 1-3, 0-5 \rangle$$
$$= \langle 8, -2, -5 \rangle.$$

#3 Find $\vec{u}\cdot\vec{v}$ if $\vec{u} = -5\vec{i} - 6\vec{k}$
$\vec{v} = \vec{i} - \vec{j} + \vec{k}$.

$$\vec{u}\cdot\vec{v} = -5\cdot 1 + 0\cdot(-1) + (-6)\cdot 1$$
$$= -5 - 6 = -11.$$

#4 Find θ... $\vec{u} = \langle 0, 2, 4 \rangle$

$\vec{v} = \langle -1, 1, 1 \rangle$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right) = \cos^{-1}\left(\frac{2+4}{\sqrt{2^2+4^2} \cdot \sqrt{1^2+1^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{6}{\sqrt{20} \cdot \sqrt{3}}\right) = \cos^{-1}\left(\sqrt{3/5}\right).$$

#5 Is $\vec{u} \perp \vec{v}$?

$\vec{u} = \vec{i} - \vec{j} + 6\vec{k}$

$\vec{v} = -\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} - 3\vec{k}$.

$\vec{u} \cdot \vec{v} = -\frac{1}{2} - \frac{1}{2} - 18 = -19 \neq 0$ (NO)

#6 What is $\text{proj}_{\vec{v}}\,\vec{u}$?

$\vec{u} = \langle 3, -4, 3 \rangle$

$\vec{v} = \langle -1, -1, 2 \rangle$

$$\left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \frac{-3+4+6}{4+1+1}\langle -1, -1, 2 \rangle$$

$$= \frac{7}{6}\langle -1, -1, 2 \rangle.$$