**Math 2110Q Worksheet 9 - Solutions**
**October 10, 2016**

1. Given $f = f(x, y, z)$ with parameterizations $x(t) = 1 - t$, $y(t) = 2t + 5$ and $z(t) = t^2$, find $f'(t = 2)$ if $\partial f/\partial x = 1$, $\partial f/\partial y = -1$ and $\partial f/\partial z = 2$ for $t = 2$. (3 pts).
**Solution:** The Chain Rule tells us that

$$f'(t) = \frac{\partial f}{\partial x}x'(t) + \frac{\partial f}{\partial y}y'(t) + \frac{\partial f}{\partial z}z'(t).$$

The values of the partial derivatives at $t = 2$ are given, so just plug them in and calculate the remaining derivatives from the given formulas for $x(t)$, $y(t)$ and $z(t)$:

$$f'(2) = x'(2) - y'(2) + 2z'(2) = -1 - 2 + 2(4) = 5.$$

2. Find the directional derivative of $f(x, y) = \ln(1 + 2x^2 + 3y^2)$ in the direction of $< -2, 2 >$ at the point $(1, 1)$. Also, calculate the maximum rate of change for $f$ at this point (4 pts).
**Solution:** We require the normalized direction vector;

$$\hat{u} = \frac{1}{\sqrt{8}} < -2, 2 > .$$

We need also $\nabla f(1, 1)$:

$$\nabla f(1, 1) = \frac{1}{1 + 2x^2 + 3y^2} < 4x, 6y > |_{(x,y)=(1,1)} = \left\langle \frac{2}{3}, 1 \right\rangle .$$

The directional derivative is now the dot-product of $\nabla f$ and $\hat{u}$, which is $1/(3\sqrt{2})$. Finally, the maximum rate of change is

$$|\nabla f(1, 1)| = \sqrt{4/9 + 1} = \frac{\sqrt{13}}{3}.$$

3. Find the equation of the tangent plane at $(x, y, z) = (2, -1, 1)$ on the surface $xy + xz + y\sin(2\pi z) = 0$. (3 pts).
**Solution:** Define $F(x, y, z) = xy + xz + y\sin(2\pi\, z)$. Then calculate

$$\nabla F =< y + z, x + \sin(2\pi\, z), x + 2\pi\, y\, \cos(2\pi\, z) > .$$

You must insert the given values $(x, y, z) = (2, -1, 1)$, yielding

$$\nabla F =< 0, 2, 2 - 2\pi > .$$

Now apply the formula for the tangent plane using the given point:

$$\nabla F\cdot < x - 2, y - (-1), z - 1 >= 0 \Rightarrow 2(y + 1) + (2 - 2\pi)(z - 1) = 0.$$