Math 2110Q Worksheet 8
October 5, 2016

1. Find the equation of the tangent plane when $(x, y) = (1, \sqrt{3}/2)$ on the surface $f(x, y) = x^2/4 + y^2$. (3 pts).
Solution: The height of the surface at the point of tangency is $f(1, \sqrt{3}/2) = z_0 = 1$. We need the partial derivatives at this point;

$$f_x = \frac{1}{2}x|_{x=1} = \frac{1}{2}, \quad f_y = 2y|_{y=\sqrt{3}/2} = \sqrt{3}.$$

Now insert these quantities into the formula for the tangent plane:

$$z - 1 = \frac{1}{2}(x - 1) + \sqrt{3}\left(y - \frac{\sqrt{3}}{2}\right).$$

2. Let the potential energy of a spring be U and $U = kx^2/2$, where k is a spring constant and x is displacement. Use differentials to estimate how large of an error in the calculation of U might exist if one can only measure $k = 2$ to within a tolerance of $\Delta k = \pm 0.02$ and $x = 1$ to within a tolerance of $\Delta x = \pm 0.01$ (3 pts.).
Solution: Approximate

$$\Delta u \approx \frac{\partial u}{\partial k}\Delta k + \frac{\partial u}{\partial x}\Delta x = \frac{1}{2}\Delta k + 2\Delta x.$$

Now insert the given tolerances. It follows that

$$\Delta u \approx \pm\left(\frac{1}{2} \cdot 0.02 + 2 \cdot 0.01\right) = \pm 0.03.$$

3. Find all second-order partial derivatives of $f(x, y) = \sin(xy)$ (3 pts).

$$\begin{aligned} f_x &= y\cos(xy) \\ f_y &= x\cos(xy) \\ f_{xx} &= -y^2\sin(xy) \\ f_{xy} &= f_{yx} = \cos(xy) - xy\sin(xy) \\ f_{yy} &= -x^2\sin(xy). \end{aligned}$$

4. Given $yz + x\ln(1 + z) = 2x^2y - 8$, we may think of $z = z(x, y)$ as a function of x and y locally near $(x, y, z) = (2, 1, 0)$. Find $\partial z/\partial x$ at the given coordinates. (3 pts.)
Solution: Use implicit differentiation as follows:

$$yz_x + \ln(1 + z) + \frac{x}{1 + z}z_x = 4xy.$$

Insert the given values $(x, y, z) = (2, 1, 0)$. The equation reduces to simply $3z_x = 8$, so that $z_x = 8/3$.