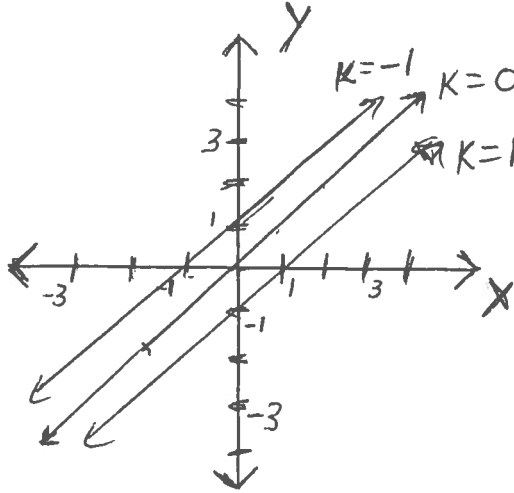


Math 2110Q Worksheet 7 Solutions
October 3, 2016

1. Sketch 3 level curves for $f(x, y) = x - y$ (2 pts.)

Solution: In other words, curves $y = x - k$ for various values of k , like $k = -1, 0, 1$.



2. Find the limit or write DNE and show why it does not exist. (2 pts. each)

$$(a) \lim_{(x,y) \rightarrow (0,0)} \frac{6x+y}{x-y} = DNE \quad (b) \lim_{(x,y) \rightarrow (0,1)} \ln \left(\frac{x^2+y}{x^2+xy+y^2} \right) = 0$$

$$(c) \lim_{(x,y) \rightarrow (-1,1)} \frac{x^2-y^2}{x+y} = -2 \quad (d) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x-y)}{x+y+1} = 0$$

For limits (b) and (d) one can cite continuity. For (c), it is necessary to note that the ratio reduces on its domain:

$$\frac{x^2 - y^2}{x + y} = \frac{(x - y)(x + y)}{x + y} = x - y.$$

For part (a), we may test the limit along lines of the form $y = kx$, $k \neq 1$. We see that the ratio in question is (for $x \neq 0$)

$$\frac{6x + y}{x - y} = \frac{6x + kx}{x - kx} = \frac{(6 + k)x}{(1 - k)x} = \frac{6 + k}{1 - k},$$

which depends on the value of k ; a limit cannot exist.