Math 2110Q Worksheet 5 - Solutions
September 19, 2016

1. Find the length of the arc $\vec{r}(t)$, $0 \leq t \leq \pi$, where $\vec{r}(t) =< \sqrt{2}\cos(t), \sqrt{2}\sin(t), 1 + 6t >$. (3 pts.)

Solution: First, calculate the length of the tangent $\vec{r}\,'(t) =< -\sqrt{2}\sin(t), \sqrt{2}\cos(t), 6 >$;

$$|\vec{r}\,'(t)| = \sqrt{2\sin^2(t) + 2\cos^2(t) + 36} = \sqrt{2 + 36} = \sqrt{38}.$$

Now, integrate to get arc length:

$$\int_0^\pi |\vec{r}\,'(t)|\ dt = \int_0^\pi \sqrt{38}\, dt = \sqrt{38}\pi.$$

2. Given a curve with vector equation $\vec{r}(t) =< e^{2t}, \ln(1 + t^2), 3 >$, find the equation of the tangent line at the point $(1, 0, 3)$. (3 pts.)

Solution: We need the direction vector for the line at the given point, for which purpose it suffices to use $\vec{r}\,'(t) =< 2e^{2t}, 2t/(1 + t^2), 0 >$. We need the value of t, which must be $t = 0$ so that $\vec{r}(0) =< 1, 0, 3 >$ points to the correct point on the line. It follows that the vector equation of the line is (in terms of a parameter, say s,)

$$< x(s), y(s), z(s) >= \vec{r}(0) + s\vec{r}\,'(0) =< 1, 0, 3 > +s < 2, 0, 0 > .$$

3. Find $\vec{T}(t)$ if $\vec{r}(t) =< 1 + t^2\sqrt{3}, t^3/3, 6t >$. (2 pts.)

Solution: First, calculate

$$\vec{r}\,'(t) =< 2t\sqrt{3}, t^2, 6 >$$

and the magnitude is $|\vec{r}\,'(t)| = \sqrt{12t^2 + t^4 + 36}$. This need not be simplified, but it can be seen that $|\vec{r}\,'(t)| = \sqrt{(t^2 + 6)^2} = t^2 + 6$. Therefore,

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{1}{t^2 + 6} < 2t\sqrt{3}, t^2, 6 > .$$

4. Let $\vec{r}(t) =< t - 1, \cos(t), \sin(t) >$. Calculate $\int_0^{\pi/2} \vec{r}(t)\, dt$. (2 pts.)

Solution: Integrate component-by-component to find the desired vector.

$$\int_0^{\pi/2} \vec{r}(t)\, dt = \left\langle \frac{1}{2}(t-1)^2, \sin(t), -\cos(t) \right\rangle |_{t=0}^{t=\pi/2} = \left\langle \frac{1}{2}\left(\left(\frac{\pi}{2} - 1\right)^2 - 1\right), 1, 1 \right\rangle.$$

5. Given a curve with parameterization $\vec{r}(t) =< 2t, t^2, 4 >$, find the rate of change of arc length when $t = 1$. (2 pts).

Solution: This is $|\vec{r}\,'(1)| = | < 2, 2, 0 > | = 2\sqrt{2}$.