Math 2110Q Worksheet 4 Solutions
September 14, 2016

1. Classify each type of **surface** below. (4 pts.)

A. $3x^2+y^2-z^2=-3$. TWO-SHEET HYPERBOLOID.

B. $y^2+7z^2=x^2$. CONE (ELLIPTIC).

C. $4x^2-y^2-3z=0$. HYPERBOLIC PARABOLOID.

D. $10y^2+z^2=2$. ELLIPTIC CYLINDER.

2. Provide examples of equations for the following four types of surfaces and label your answers by surface type: one-sheet hyperboloid, ellipsoid, paraboloid and plane. (4 pts.)
Solution: There are infinitely many correct answers, but here are simple examples.

- 1-sheet hyperboloid: $x^2+y^2-z^2=1$.
- Ellipsoid: $x^2+y^2+z^2=1$.
- Paraboloid: $z=x^2+y^2$.
- Plane: $x+y+z=1$.

3. Parameterize the curve formed by the intersection of the surfaces $-x+y+z=1$ and $2y+3z^2=5$. (4 pts.)
Solution: A curve requires a single parameter, which you must identify by inspection of the equations. In the second equation, we easily determine

$$y=y(z)=\frac{5-3z^2}{2}.$$

From the first equation, it follows that

$$x=y+z-1=y(z)+z-1=\frac{5-3z^2}{2}+z-1.$$

Now we have both $x=x(z)$ and $y=y(z)$ in terms of the single parameter z. This parameterizes the curve of intersection.

4. Given that $\vec{r}(t)=<t^2-1,\cos(\pi t),t/(t+1)>$, find

$$\lim_{t\to 1}\vec{r}(t) \text{ and } \lim_{t\to -1}\vec{r}(t) \text{ (2 pts.)}$$

Solution: The only trickly part is to realize that

$$\lim_{t\to -1}\frac{t}{t+1}$$

does not exist, so that $\lim_{t\to -1}\vec{r}(t)$ does not exist. On the other hand, we see that

$$\lim_{t\to 1}\vec{r}(t)=<0,-1,1/2>.$$