Math 2110Q Worksheet 3 Solutions
September 12, 2016

1. Given $\vec{u} = < 2, -1, -1 >$, find the equation(s) for the line that passes through $(0, 4, -5)$ in the direction $\vec{u}$. (2 pts.)
Solution: In vector form (any form acceptable) we could write

$$\vec{r}(t) = < 0, 4, -5 > + t < 2, -1, -1 > .$$

2. Find a (non-zero) vector perpendicular to both $\vec{u} = < 3, 0, -1 >$ and $\vec{v} = < -1, 2, -3 >$. (3 pts.)
Solution: One takes the cross-product, for example, to generate the orthogonal vector

$$\vec{u} \times \vec{v} = < 2, 10, 6 > .$$

Any nontrivial multiple of this is correct.

3. Given a line with symmetric equations

$$\frac{x-1}{3} = \frac{y+2}{5} \text{ and } z = -7,$$

find the corresponding vector equation for the line. (3 pts.)
Solution: Set the two fractions equal to a parameter t and solve for x and y in terms of t. Then we see the vector equation must be

$$\vec{r}(t) = < x(t), y(t), z(t) > = < 1 + 3t, -2 + 5t, -7 > .$$

4. Find the equation(s) for the plane containing the points $A = (2, -2, 0)$, $B = (3, 0, 0)$ and $C = (0, 0, 1)$. (4 pts.)
Solution: Get a normal for the plane by calculating vectors $\vec{AB}$ and $\vec{AC}$, then use $\vec{n} = \vec{AB} \times \vec{AC}$. Any multiple of this works, but

$$\vec{n} = < 1, 2, 0 > \times < -2, 2, 1 > = < 2, -1, 6 > .$$

Any of the points in the plane may be inserted into the vector equation for the plane. For example, using point A the equation is

$$< 2, -1, 6 > \cdot < x - 2, y + 2, z > = 0.$$

This is sufficient; other forms are also fine.

5. Given vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ with $|\vec{u}| = 2$, $|\vec{v}| = 3$ and an acute angle in between them of $\theta = \pi/6$, calculate $|\vec{u} \times \vec{v}|$ (2 pts.)
Solution: Just recall that

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin(\theta) = 6 \sin(\pi/6) = 3.$$

The last reduction to 3 is not required for credit here.