Math 2110Q Worksheet 2-Solutions
September 7, 2016

1. Find a vector of length 2 that points in the direction $-\vec{i}+\vec{j}+2\vec{k}$. (2 pts.)
Solution: First, NORMALIZE the vector to have length 1. The magnitude of the given vector is $\sqrt{(-1)^2+(1)^2+(2)^2}=\sqrt{6}$. A unit vector in the same direction is thus

$$\frac{1}{\sqrt{6}}\left(-\vec{i}+\vec{j}+2\vec{k}\right)=-\frac{1}{\sqrt{6}}\vec{i}+\frac{1}{\sqrt{6}}\vec{j}+\frac{2}{\sqrt{6}}\vec{k}.$$

Now, we can take this vector and multiply by the desired length, which is 2, to get the answer:

$$-\frac{2}{\sqrt{6}}\vec{i}+\frac{2}{\sqrt{6}}\vec{j}+\frac{4}{\sqrt{6}}\vec{k}.$$

(NOTE: I don't care about "reducing" or "simplifying" the coefficients.)

2. Let $\vec{u}=<7,0,-9>$ and $\vec{v}=<3,-2,1>$. Find $\vec{u}\cdot\vec{v}$. (2 pts.)
Solution: Use the formula as follows:

$$\vec{u}\cdot\vec{v}=(7)(3)+(0)(-2)+(-9)(1)=12.$$

3. Find a vector $\vec{v}$ orthogonal to $\vec{u}=<1,-1,2>$ such that $|\vec{v}|\neq 0$. There is no unique answer. (2 pts.)
Solution: This is related to "linear algebra", by the way. Let us propose that $\vec{v}=<v_1,v_2,v_3>$. Then we require simply that

$$\vec{u}\cdot\vec{v}=0\Rightarrow v_1-v_2+2v_3=0.$$

EVERY choice of two numbers v_1 and v_2 (not both zero) yields a solution by then solving for v_3 from

$$v_3=\frac{1}{2}v_2-\frac{1}{2}v_1.$$

For example, if $v_1=v_2=1$ then $v_3=0$, hence $\vec{v}=<1,1,0>$ works.

4. Find the angle between $\vec{u}=<1,\sqrt{11},2>$ and $\vec{k}$. (3 pts.)
Solution: First, recall that $\vec{k}=<0,0,1>$. Then use the formula

$$\theta=\cos^{-1}\left(\frac{\vec{u}\cdot\vec{k}}{|\vec{u}||\vec{k}|}\right)=\cos^{-1}\left(\frac{(1)(0)+(\sqrt{11})(0)+(2)(1)}{\left(\sqrt{1^2+(\sqrt{11})^2+(2)^2}\right)(1)}\right)$$
$$=\cos^{-1}\left(\frac{2}{\sqrt{1+11+4}}\right)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}.$$

One could answer 60° or just leave it as $\cos^{-1}(0.5)$.

5. Find the projection of $2\vec{i}+3\vec{j}-\vec{k}$ onto $\vec{i}+\vec{j}$. (3 pts.)
Solution: Think of it this way:

$$\vec{u}=<2,3,-1>$$
$$\vec{v}=<1,1,0>$$
$$\text{proj}_{\vec{v}}(\vec{u})=\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v}=\frac{2+3}{1+1}<1,1,0>=\frac{5}{2}<1,1,0>.$$

6. A vector $\vec{u} = < u_1, u_2 >$ has length 2 and if the tail is at the origin, it makes a (positive) 150° angle with respect to the x-axis. Calculate the components u_1 and u_2. (3 pts.)
Solution: One envisions that the vector $\vec{u}$ has its tail at the origin and points to (u_1, u_2) on a circle of radius 2. Trigonometry tells us that the coordinates (and hence components of $\vec{u}$) are thus

$$u_1 = 2\cos(150^\circ) \text{ and } u_2 = 2\sin(150^\circ),$$

which reduces (did not require for credit) to $\vec{u} = < -\sqrt{3}, 1 >$.