Math 2110Q Worksheet 18 Solutions
November 30, 2016

1. Calculate $\nabla \times \vec{F}$ and $\nabla \cdot \vec{F}$ for the vector field $\vec{F}(x,y,z) =< 1, z^2, 2yz >$. Determine if $\vec{F}$ is a conservative vector field.

Solution:

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & z^2 & 2yz \end{vmatrix} =< 0,0,0 > .$$

It follows that the field is conservative. The divergence is

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(1) + \frac{\partial}{\partial y}(z^2) + \frac{\partial}{\partial z}(2yz) = 2y.$$

2. Provide a parameterization for the surface $\{(x,y,z) \,|\, x^2+y^2+z^2 = 4\}$.

Solution: This is just a sphere of radius 2, so use spherical coordinates with $\rho = 2$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$ and

$$x = 2\cos(\theta)\sin(\phi),\ y = 2\sin(\theta)\sin(\phi),\ z = 2\cos(\phi).$$

3. Let $\vec{r}(u,v) =< (1+u)\cos(v), (1+u)\sin(v), u >$ for parameters $0 \le u \le 1$ and $0 \le v \le 2\pi$. Find the area of this parametric surface.

Solution: First, calculate

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \cos(v) & \sin(v) & 1 \\ -(1+u)\sin(v) & (1+u)\cos(v) & 0 \end{vmatrix}$$
$$=< -(1+u)\cos(v), -(1+u)\sin(v), 1+u >= (1+u) < -\cos(v), -\sin(v), 1 > .$$

Next, note that $|\vec{r}_u \times \vec{r}_v| = (1+u)\sqrt{2}$. The surface area is thus

$$\sqrt{2}\int_0^{2\pi}\int_0^1 1+u\,du\,dv = \frac{3\sqrt{2}}{2}\int_0^{2\pi} dv = 3\pi\sqrt{2}.$$