

Math 2110Q Worksheet 17 Solutions
November 28, 2016

1. Determine if the following vector fields are conservative. You must justify your answer for credit.

(a) $\vec{F}(x, y) = \langle y + 1, x + 2y \rangle$

Solution: Just check as follows:

$$\frac{\partial}{\partial x}(x + 2y) - \frac{\partial}{\partial y}(y + 1) = 1 - 1 = 0 \Rightarrow \text{conservative.}$$

(b) $\vec{F}(x, y) = \langle 2ye^{2x+y}, (1+y)e^{2x+y} + x \rangle$

Solution:

$$\frac{\partial}{\partial x}((1+y)e^{2x+y} + x) - \frac{\partial}{\partial y}(2ye^{2x+y}) = 2(1+y)e^{2x+y} + 1 - (2+2y)e^{2x+y} = 1 \Rightarrow \text{not conservative.}$$

2. Find a potential function for the (conservative) vector field $\vec{F}(x, y) = \langle 4x, 2y \rangle$. (4 pt.)

Solution: We seek $f = f(x, y)$ such that $\nabla f = \vec{F}$. It follows that $f_x = 4x$, so $f = 2x^2 + g(y)$, for some unknown function $g(y)$. To solve for $g(y)$, we note $f_y = 2y = g'(y)$, so that $g(y) = y^2 + c$, for any constant c . Thus, we may take $f(x, y) = 2x^2 + y^2 + c$.

3. Let C be the curve that bounds the 2D region

$$\mathcal{D} = \{(x, y) \mid 0 \leq x \leq 1, -\sqrt{x} \leq y \leq \sqrt{x}\},$$

with C oriented positively with respect to \mathcal{D} . Calculate

$$\int_C 4y dx + 7x dy.$$

Solution: The easy way is to apply Green's Theorem and convert the integral into a double integral:

$$\int_C 4y dx + 7x dy = \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \left(\frac{\partial}{\partial x}(7x) - \frac{\partial}{\partial y}(4y) \right) dy dx = 3 \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} dy dx = 6 \int_0^1 \sqrt{x} dx = 4.$$