Math 2110Q Worksheet 16 Solutions
November 16, 2016

1. Let $\vec{F}(x,y,z) =< 1,1,1 >$, a constant vector field. Show that this vector field is conservative. *Hint: consider* $f(x,y,z) = x+y+z$.

Solution: One need only point out that $\nabla f = \vec{F}$, so the vector field is conservative.

2. Let a thin, solid wire have mass density $\rho(x,y) = 2x$, where the wire has position in 2D space given by the part of the curve $y = x^2$ with $1 \leq x \leq 2$. Calculate the mass of the wire.

Solution: A simple parameterization of the wire segment is $\vec{r}(x) =< x, x^2 >$, for $1 \leq x \leq 2$. Then $\vec{r}'(x) =< 1, 2x >$ and the mass is given by

$$\int_1^2 2x\sqrt{1+4x^2}\,dx = \frac{1}{4}\int_5^{17} \sqrt{u}\,du = \frac{1}{6}\left(17^{3/2} - 5^{3/2}\right).$$

3. A particle moves along the circular path $x^2 + y^2 = 1$ starting at $(1,0)$ and proceeding counter-clockwise to $(-1,0)$. If a force $\vec{F}(x,y) =< -y, x >$ acts on the particle, calculate the work done.

Solution: We can parameterize the curve using the idea of polar coordinates; $\vec{r}(t) =< \cos(t), \sin(t) >$, for $0 \leq t \leq \pi$. Then $\vec{r}'(t) =< -\sin(t), \cos(t) >$ and it follows that the work done is

$$\int_0^{\pi} F(\vec{r}(t)) \cdot \vec{r}'(t)\,dt = \int_0^{\pi} < -\sin(t), \cos(t) > \cdot < -\sin(t), \cos(t) > dt = \int_0^{\pi} \sin^2(t) + \cos^2(t)\,dt = \int_0^{\pi} dt = \pi.$$