Math 2110Q Worksheet 12
October 26, 2016

1. Calculate the volume under the surface $z = 4 - x^2 - y^2$ and above the xy-plane. (3 pts.)
Solution: Note that the intersection of the surface with the xy-plane is the circle $x^2 + y^2 = 4$, which has radius 2. Use polar:

$$\int_0^{2\pi} \int_0^2 \left(4 - r^2\right) r \, dr \, d\theta = 2\pi \int_0^2 \left(4r - r^3\right) \, dr = 2\pi \left(2r^2 - \frac{1}{4}r^4\right) |_0^2$$
$$= 2\pi \left(8 - 4\right) = 8\pi.$$

2. Let T be the area enclosed by the parallelogram with vertices $\{(0,0), (1,0), (1,1), (2,1)\}$. Calculate the value of the following integral. (4 pts.)

$$\int\int_T \frac{1}{1+y} \, dA.$$

Solution: This can be set up in different ways, but probably it is easiest to integrate in the x-direction first. The reason is that y is neatly bounded on the given domain between 0 and 1, but we may express x as being bounded as $y \leq x \leq 1 + y$. The integral is then

$$\int_0^1 \int_y^{1+y} \frac{1}{1+y} \, dx \, dy = \int_0^1 \frac{x}{1+y}\bigg|_y^{1+y} \, dy = \int_0^1 \frac{1}{1+y} \, dy = \ln(1+y)|_0^1 = \ln(2).$$

3. A curve in the xy-plane, described using polar coordinates, has equation $r^2 = \sin(\theta)$, $0 \leq \theta \leq \pi$. Find the area of the region enclosed by the curve. (3 pts.)
Solution: The key is to note that r is bounded between 0 and $\sqrt{\sin(\theta)}$. Then the area, using polar coordinates, is

$$\int_0^{\pi} \int_0^{\sqrt{\sin(\theta)}} r \, dr \, d\theta = \frac{1}{2} \int_0^{\pi} \sin(\theta) \, d\theta = -\frac{1}{2} \left(\cos(\pi) - \cos(0)\right) = 1.$$