Math 2110Q Worksheet 11 Solutions
October 24, 2016

1. Find the volume of the region bounded by the following surfaces: $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = x^2 + y^2$ (4 pts).
Solution: This may be thought of as the volume under the surface $f(x, y) = x^2 + y^2$, above the square region of the xy-plane with $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Thus, the volume is given by the double integral

$$\begin{aligned}\int_0^1 \int_0^1 x^2 + y^2 \, dx \, dy &= \int_0^1 \left(\frac{1}{3}x^3 + xy^2 \right) |_0^1 \, dy = \int_0^1 \frac{1}{3} + y^2 \, dy \\ &= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}.\end{aligned}$$

2. Given $f(x, y) = y \sin(xy)$ over a domain $\mathcal{D} = \{(x, y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq \pi/2\}$, calculate (4 pts.)

$$\int \int_{\mathcal{D}} f(x, y) \, dA.$$

Solution: Write as an iterated integral, but it is easiest to do it in the following order:

$$\begin{aligned}\int_0^{\pi/2} \int_0^1 y \sin(xy) \, dx \, dy &= \int_0^{\pi/2} \int_0^1 \frac{\partial}{\partial x} - \cos(xy) \, dx \, dy = \int_0^{\pi/2} - \cos(xy)|_0^1 \, dy \\ &= \int_0^{\pi/2} 1 - \cos(y) \, dy = (y - \sin(y))|_0^{\pi/2} = \frac{\pi}{2} - 1.\end{aligned}$$