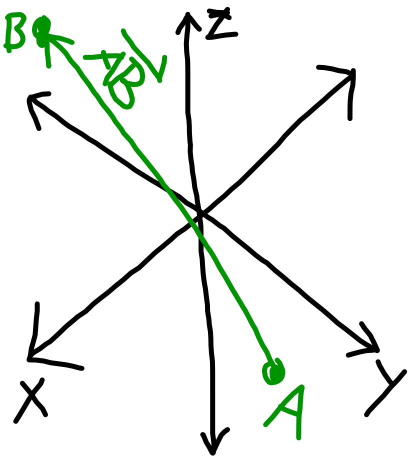


$\overrightarrow{AB}$ points from A to B

$A(a_1, a_2, a_3)$

$B(b_1, b_2, b_3)$

$\Rightarrow \overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle$

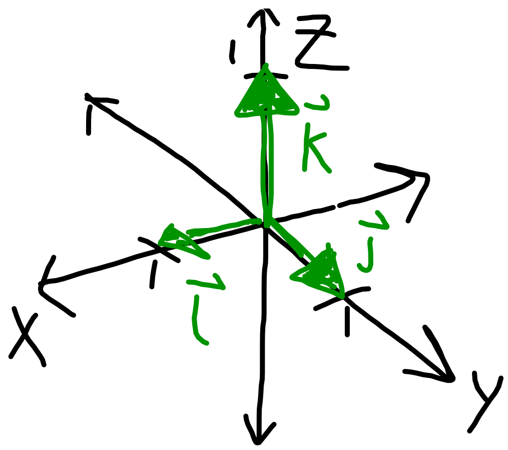
Ex: Let $A(0,2,-1)$ & $B(1,10,3)$.

Then $\overrightarrow{AB} = \langle 1-0, 10-2, 3+1 \rangle = \langle 1, 8, 4 \rangle$.

$\overrightarrow{BA} = \langle 0-1, 2-10, -1-3 \rangle = \langle -1, -8, -4 \rangle = -\overrightarrow{AB}$.

Standard basis vectors

$\vec{i} = \langle 1, 0, 0 \rangle$
$\vec{j} = \langle 0, 1, 0 \rangle$
$\vec{k} = \langle 0, 0, 1 \rangle$



Note that we may write

$$\langle a, b, c \rangle = a\langle 1,0,0\rangle + b\langle 0,1,0\rangle + c\langle 0,0,1\rangle$$
$$= a\vec{i} + b\vec{j} + c\vec{k}$$

EX: $\langle -5, 6, 4\rangle = -5\vec{i} + 6\vec{j} + 4\vec{k}$.

Normalizing vectors $\quad \hat{v} = \frac{1}{|\vec{v}|}\vec{v}$. "unit vector"

$$|\hat{v}| = \left|\frac{1}{|\vec{v}|}\vec{v}\right| = \frac{1}{|\vec{v}|}|\vec{v}| = 1.$$

EX: Find a vector of length 2 in the direction

of < 1, 1, -1 >. Let $\vec{v} = \langle 1, 1, -1\rangle$; we need $2\hat{v}$...

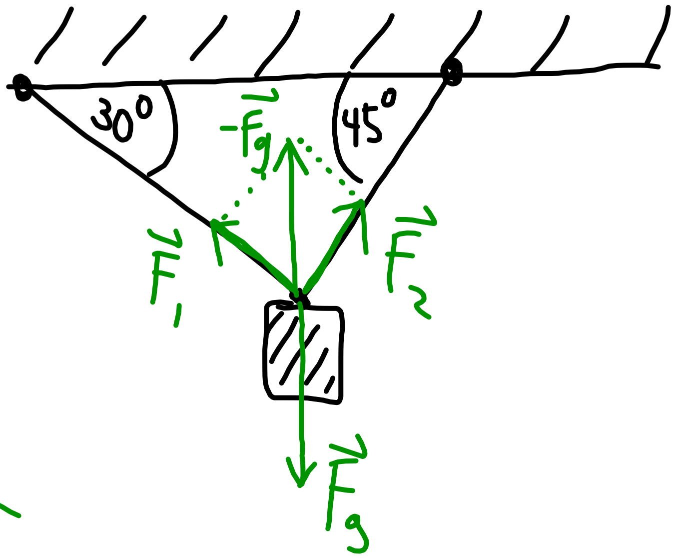
$$\frac{2}{|\vec{v}|}\vec{v} = \frac{2}{\sqrt{3}}\langle 1, 1, -1\rangle.$$

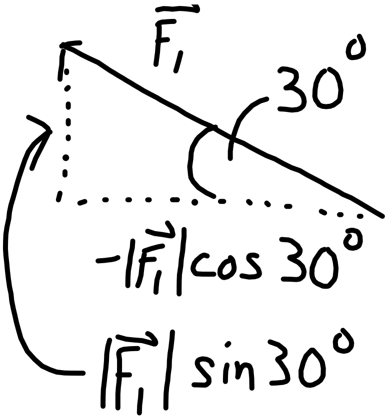
EX: Find $\hat{v}$ if $\vec{v} = \vec{j} - 2\vec{k}$.

$$\hat{v} = \frac{1}{\sqrt{5}}\left(\vec{j} - 2\vec{k}\right) = \frac{1}{\sqrt{5}}\vec{j} - \frac{2}{\sqrt{5}}\vec{k}.$$

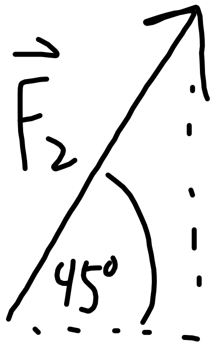
APPLICATION: If something is at rest, the net force is zero...

$$\vec{F}_1 + \vec{F}_2 = -\vec{F}_g$$





$$\vec{F}_1 = \left(-|\vec{F}_1|\cos 30^\circ\right)\vec{i} + \left(|\vec{F}_1|\sin 30^\circ\right)\vec{j}$$



$$\vec{F}_2 = |\vec{F}_2|\cos 45^\circ\,\vec{i} + |\vec{F}_2|\sin 45^\circ\,\vec{j}.$$

Recall $\vec{F}_1 + \vec{F}_2 = -\vec{F}_g = 20\vec{j}$ (example)

Trig... $\vec{F}_1 = -|\vec{F}_1|\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}|\vec{F}_1|\vec{j}$

$\vec{F}_2 = \frac{1}{\sqrt{2}}|\vec{F}_2|\vec{i} + \frac{1}{\sqrt{2}}|\vec{F}_2|\vec{j}$

$\Rightarrow \left(\frac{1}{\sqrt{2}}|\vec{F}_2| - |\vec{F}_1|\frac{\sqrt{3}}{2}\right)\vec{i} + \left(\frac{1}{\sqrt{2}}|\vec{F}_2| + \frac{1}{2}|\vec{F}_1|\right)\vec{j}$

$= 20\vec{j} + 0\vec{i}$

$0 = \frac{1}{\sqrt{2}}|\vec{F}_2| - |\vec{F}_1|\frac{\sqrt{3}}{2}$ ... $20 = \frac{1}{\sqrt{2}}|\vec{F}_2| + \frac{1}{2}|\vec{F}_1|$

$$|\vec{F_1}|\frac{1}{2}+\frac{1}{\sqrt{2}}|\vec{F_2}|=20$$

$$-\frac{\sqrt{3}}{2}|\vec{F_1}|+\frac{1}{\sqrt{2}}|\vec{F_2}|=0 \Rightarrow |\vec{F_2}|=\frac{\sqrt{6}}{2}|\vec{F_1}|$$

$$\Rightarrow \frac{1}{2}|\vec{F_1}|+\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{6}}{2}|\vec{F_1}|=\frac{1}{2}(1+\sqrt{3})|\vec{F_1}|=20$$

$$\Rightarrow |\vec{F_1}|=\frac{40}{1+\sqrt{3}}$$

(Insert above to get $|\vec{F_2}|$).

DOT PRODUCTS $\vec{u}\vec{v}$?? (meaningless)

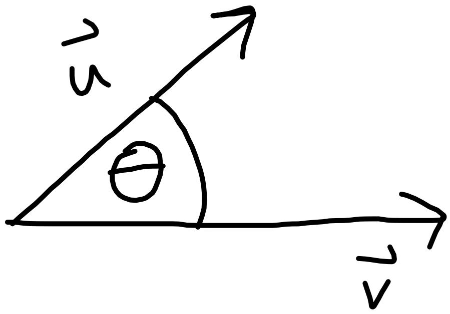
DOT: $\boxed{\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3}$

PROPERTIES: ① $|\vec{u}|^2 = \vec{u}\cdot\vec{u}$

② $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$

③ $\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$

④ $(c\vec{u})\cdot\vec{v} = c(\vec{u}\cdot\vec{v}) = \vec{u}\cdot(c\vec{v})$



$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right).$$

Ex: $\vec{u} = \langle 1,1,1\rangle \quad \vec{v} = \langle 2,0,1\rangle$

Find θ ...

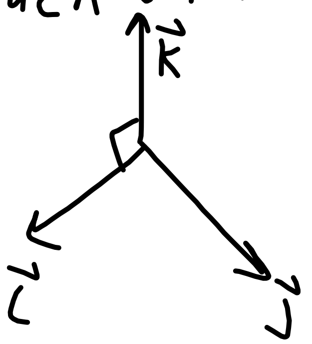
$$\theta = \cos^{-1}\left(\frac{2+0+1}{\sqrt{3}\cdot\sqrt{5}}\right)$$

<u>Orthogonal</u> vectors lie at right angles to each other.

"$\vec{u} \perp \vec{v}$"

"$\vec{u}$ is orthogonal to $\vec{v}$"

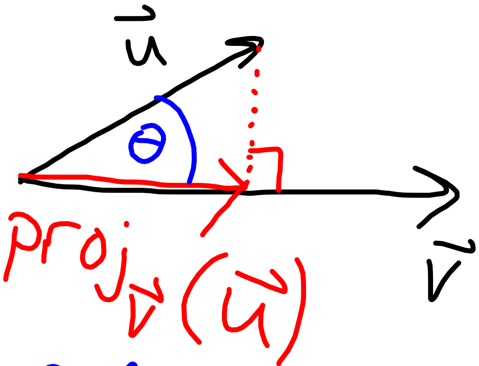
EX:



$$\cos 90^\circ = 0 = \vec{u}\cdot\vec{v}$$

EX: $\vec{i}\cdot\vec{j} = \langle 1,0,0\rangle \cdot \langle 0,1,0\rangle = 0$

Projections



$$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\,\hat{v}$$

$$= |\vec{u}|\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\hat{v} = \underbrace{\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}}_{\text{comp}_{\vec{v}}(\vec{u})}\hat{v}$$

$$\boxed{= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v}}$$

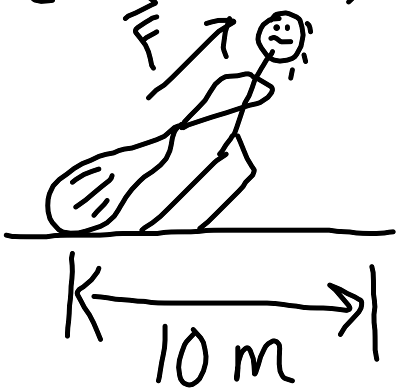
EX: $\vec{u} = \langle 2, 3, -4 \rangle$, $\vec{v} = \langle 0, 1, -1 \rangle$...

Find $\text{proj}_{\vec{v}}\,\vec{u}$.

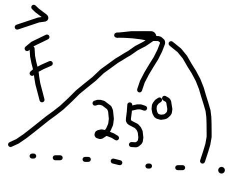
$$\text{proj}_{\vec{v}}\,\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v} = \frac{3+4}{2}\langle 0, 1, -1 \rangle$$

$$= \left\langle 0, \frac{7}{2}, -7/2 \right\rangle$$

Calculation of the work done by a constant force.



$|\vec{F}| = 90N$



$$\vec{F} = 90\cos 25^\circ \,\vec{i} + 90\sin 25^\circ \,\vec{j}$$

$$\vec{D} = 10\vec{i}$$

$$W = \vec{F}\cdot\vec{D} = 900\cos 25^\circ$$

#1 Let $\vec{u} = \vec{i} + 2\vec{j} - 4\vec{k}$.

(a) Find $\hat{u}$. $|\vec{u}| = \sqrt{21}$

$$\hat{u} = \frac{1}{\sqrt{21}}\vec{u} = \frac{1}{\sqrt{21}}\vec{i} + \frac{2}{\sqrt{21}}\vec{j} - \frac{4}{\sqrt{21}}\vec{k}$$

(b) Find a vector of length 5 in the opposite direction of $\vec{u}$.

$$-5\hat{u} = \frac{-5}{\sqrt{21}}\vec{i} - \frac{10}{\sqrt{21}}\vec{j} + \frac{20}{\sqrt{21}}\vec{k}.$$

#2 Find $\overrightarrow{AB}$; $A=(-2,3,5)$, $B=(6,1,0)$.

$\langle 6-(-2), 1-3, 0-5 \rangle$

$= \langle 8, -2, -5 \rangle$.

#3 Find $\vec{u}\cdot\vec{v}$ if $\vec{u} = -5\vec{i} - 6\vec{k}$

$\vec{v} = \vec{i} - \vec{j} + \vec{k}$.

$$\vec{u}\cdot\vec{v} = -5\cdot 1 + 0\cdot(-1) - 6\cdot 1$$

$$= -5 - 6 = -11$$

$$\vec{u} = \langle -5, 0, -6 \rangle$$

$$\vec{v} = \langle 1, -1, 1 \rangle$$

#4 Find θ ... $\vec{u} = \langle 0, 2, 4 \rangle$

$\vec{v} = \langle -1, 1, 1 \rangle$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right) = \cos^{-1}\left(\frac{6}{\sqrt{20} \cdot \sqrt{3}}\right)$$

#5 Is $\vec{u} \perp \vec{v}$?

$\vec{u} = \vec{i} - \vec{j} + 6\vec{k}$

$\vec{v} = -\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} - 3\vec{k}$.

$\vec{u} \cdot \vec{v} = -19 \neq 0$ (NO)

#6 What is $\text{proj}_{\vec{v}}\,\vec{u}$?

$\vec{u} = \langle 3, -4, 3 \rangle$

$\vec{v} = \langle -1, -1, 2 \rangle$

$$\left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \frac{-3+4+6}{4+1+1}\langle -1, -1, 2 \rangle$$

$$= \frac{7}{6}\langle -1, -1, 2 \rangle.$$