Exam 1 Review Solutions

#1 $\vec{r}(t) = \vec{r}_0 + t\,\vec{v}$

$$\vec{r}_0 = \langle 3, -3, 1 \rangle$$

$$\vec{v} = \langle 1, 1, 2 \rangle$$

#2 $\vec{r}_1 = \langle -2+t, 1-t, 2t \rangle$

$\vec{r}_2 = \langle 5t-2, 2t+1, t \rangle$

$\vec{v}_1 = \langle 1, -1, 2 \rangle$

$\vec{v}_2 = \langle 5, 2, 1 \rangle$

$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \frac{5-2+2}{\sqrt{6}\cdot\sqrt{30}} = \frac{5}{6\sqrt{5}} = \frac{\sqrt{5}}{6}.$$

$$\boxed{\theta = \cos^{-1}\left(\sqrt{5}/6\right)}$$

$\vec{v}_1 \times \vec{v}_2 = \vec{n} = \langle -5, 9, 7 \rangle$ (plane normal)

Take $t=0$ in $\vec{r}_1$... get point $(-2, 1, 0)$

$$\vec{n} \cdot \langle x-(-2), y-1, z-0 \rangle = 0$$

$$\boxed{\Rightarrow -5(x+2) + 9(y-1) + 7z = 0}$$

#3 $|\vec{r}\,'| = |\langle 2, 2t, t^2 \rangle|$

$= \sqrt{2^2 + (2t)^2 + t^4} = \sqrt{4 + 4t^2 + t^4} = \sqrt{(t^2+2)^2}$

$= t^2 + 2$

Note $\vec{r}(0) = \langle 0, 0, 0 \rangle$ $(t=0)$

$\vec{r}(2) = \langle 4, 4, 8/3 \rangle$ $(t=2)$

$S = \int_0^2 t^2 + 2 \, dt = \frac{8}{3} + 4 = \boxed{\frac{20}{3}}$.

#4

(a) one-sheet hyperboloid

(b) elliptic cylinder

(c) ellipsoid

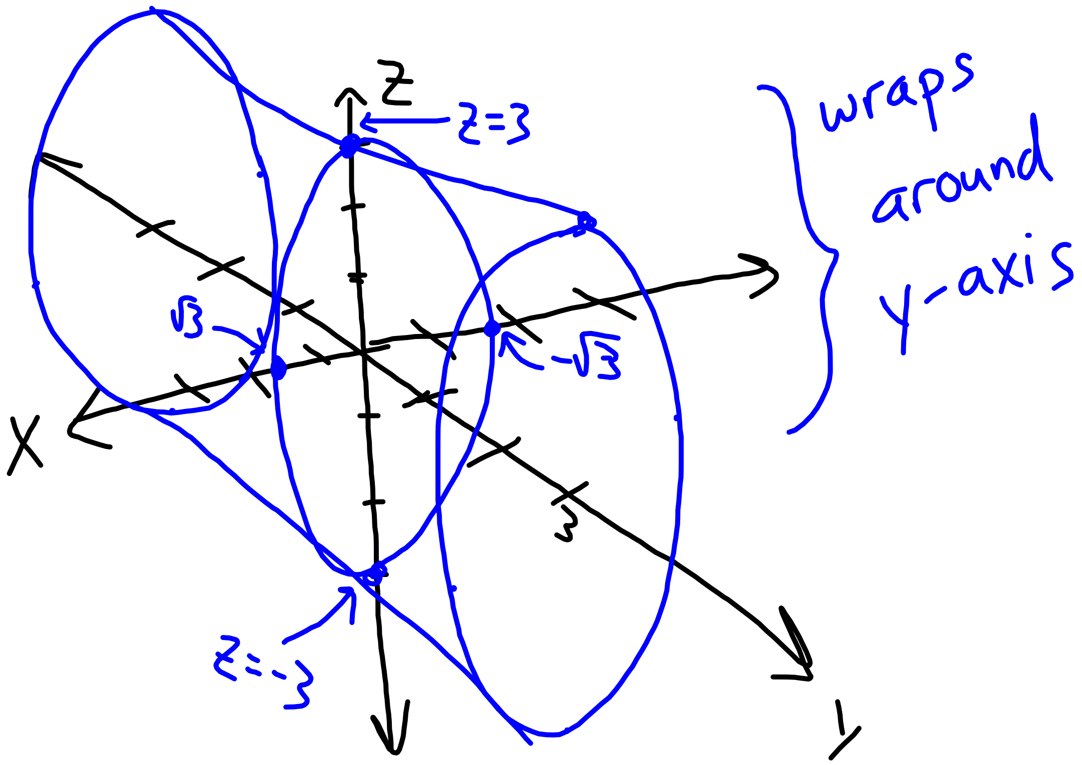
(d) hyperbolic paraboloid

(e) two-sheet hyperboloid

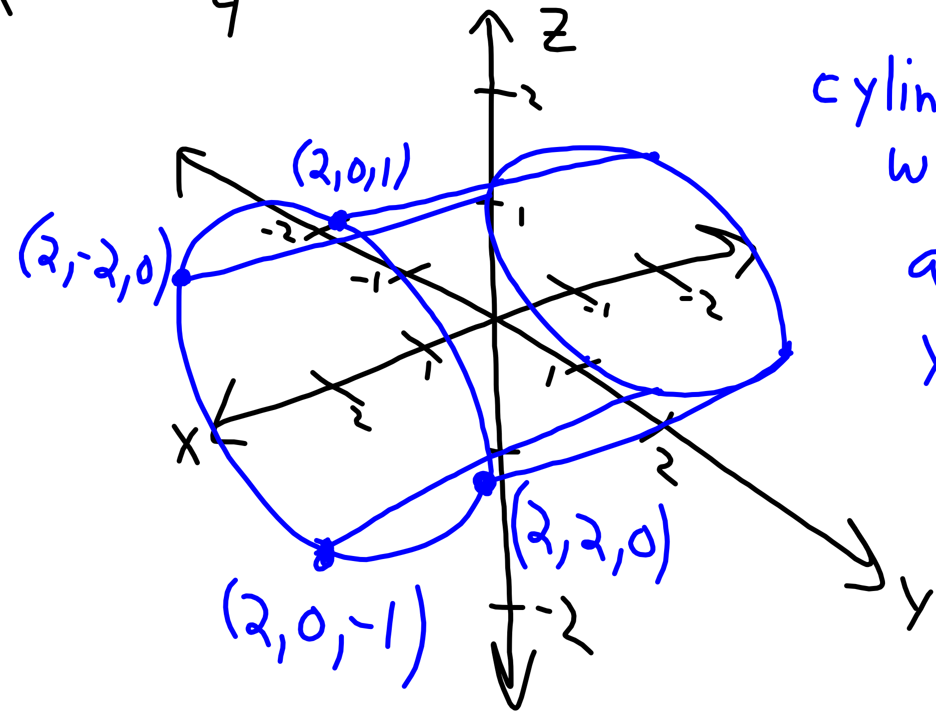
(f) elliptic paraboloid

(g) cone

(4a) $\frac{x^2}{3} - \frac{y^2}{9} + \frac{z^2}{9} = 1$

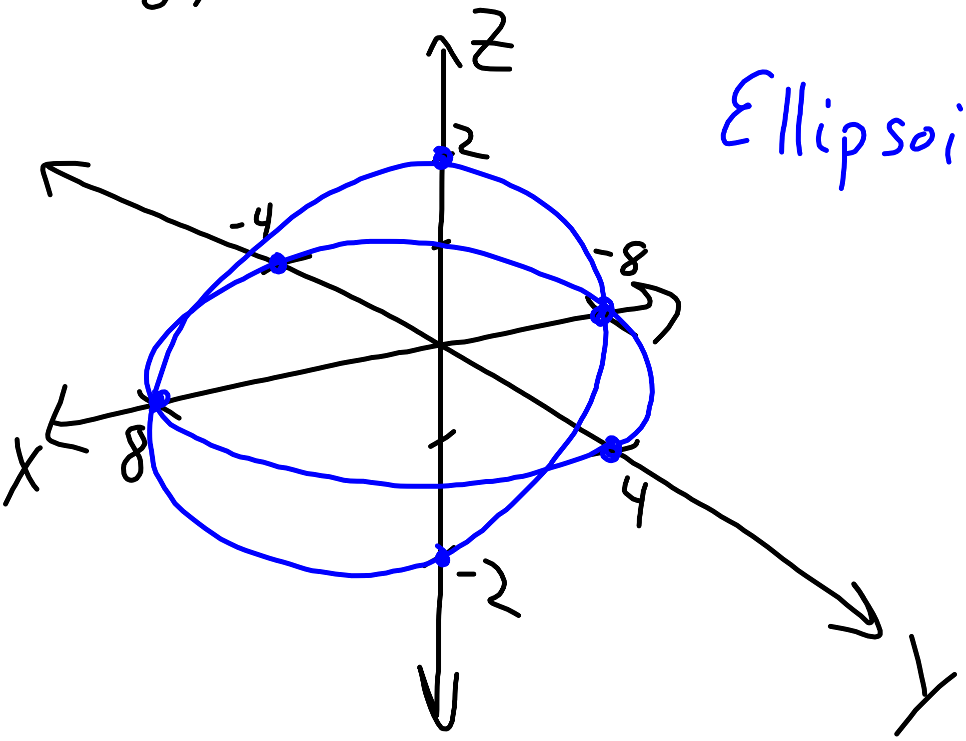


(4b) $\frac{y^2}{4}+z^2=1$




cylinder wraps around x-axis

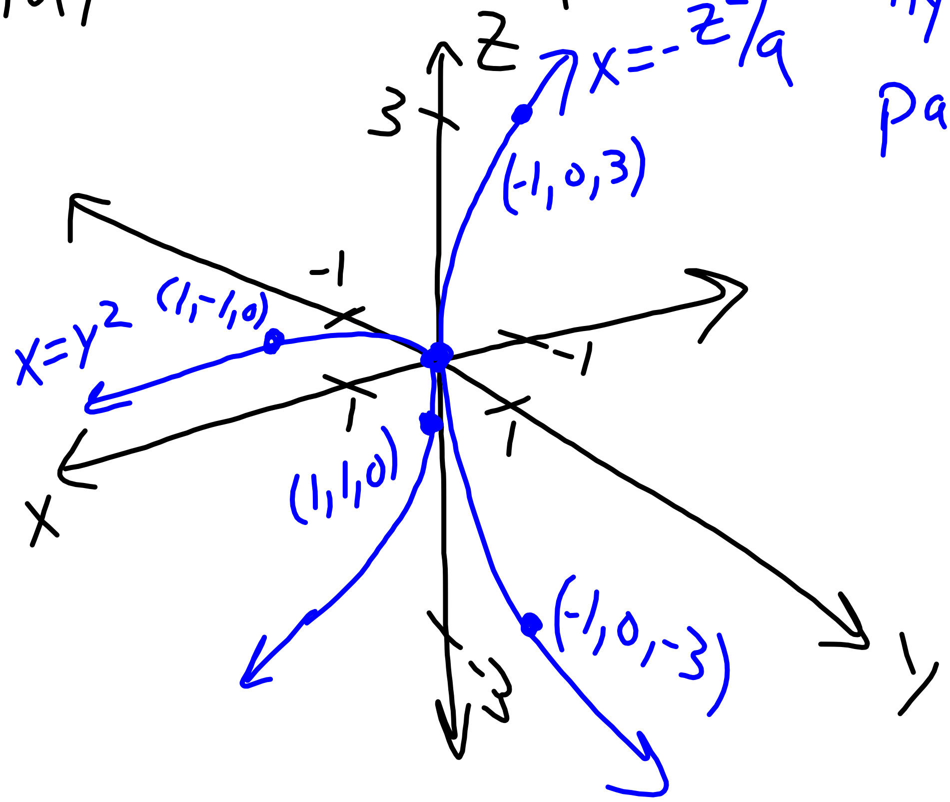
(4c) $\left(\frac{x}{8}\right)^2 + \left(y/4\right)^2 + \left(\frac{z}{2}\right)^2 = 1$



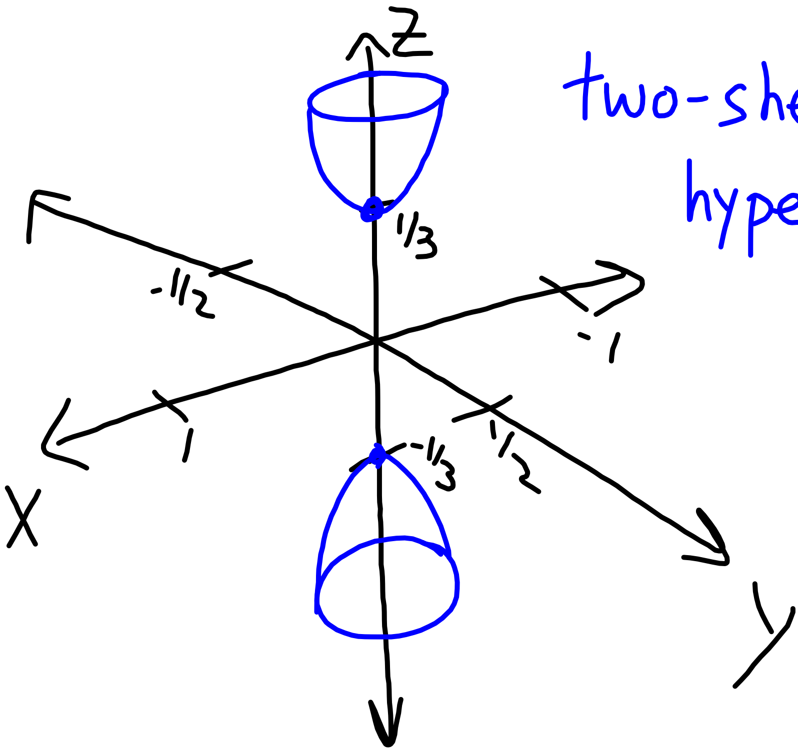

Ellipsoid

(4d) $x = y^2 - z^2/9$

Hyperbolic paraboloid

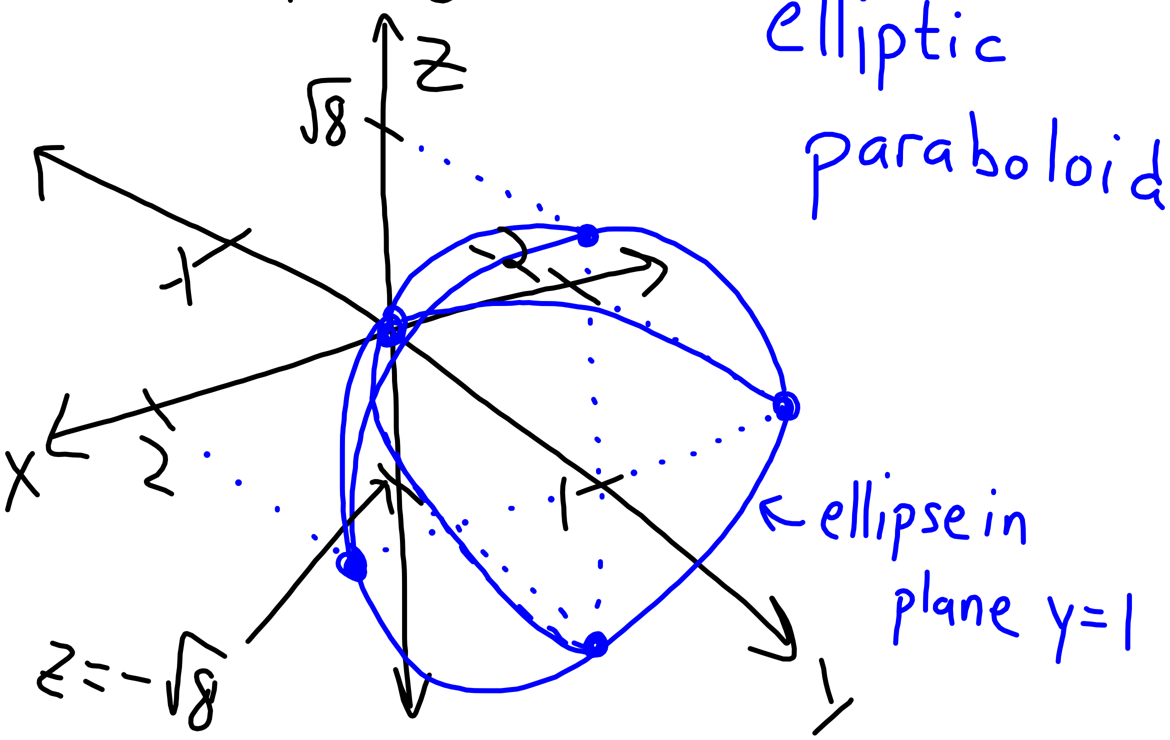


(4e) $\left(\frac{z}{1/3}\right)^2 - \left(\frac{y}{1/2}\right)^2 - x^2 = 1$

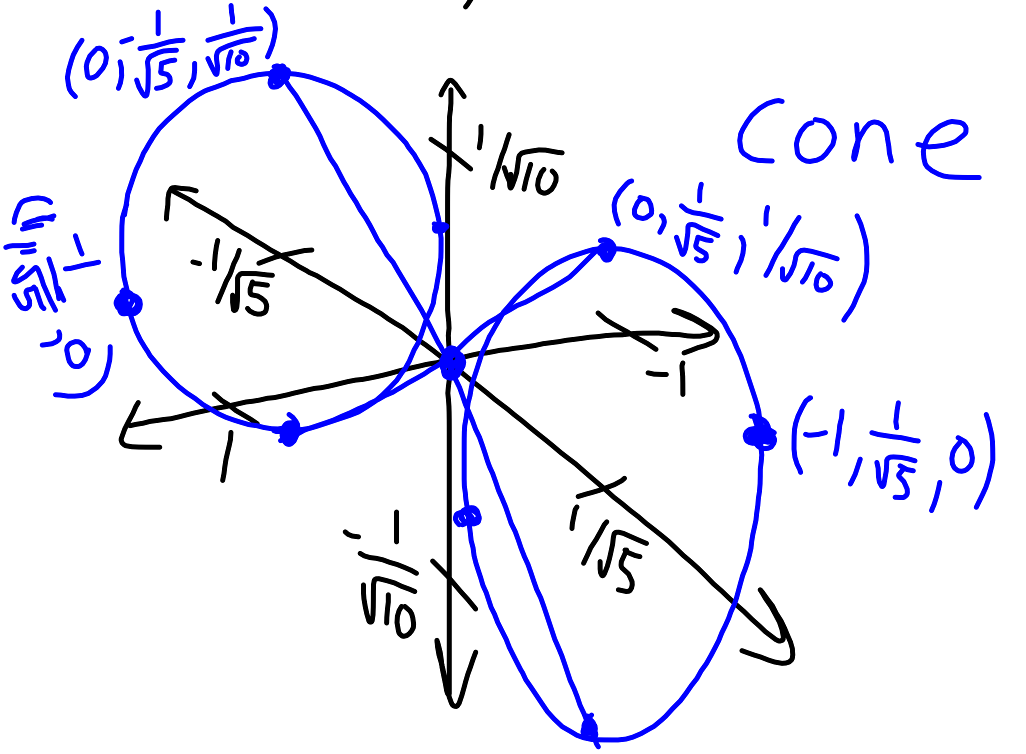



two-sheet hyperboloid

(4 f) $y = \frac{x^2}{4} + \frac{z^2}{8}$



(4g) $\left(\frac{y}{1/\sqrt{5}}\right)^2 = x^2 + \left(\frac{z}{1/\sqrt{10}}\right)^2$



#5 $\vec{r}\,' = \langle -\sqrt{2}\sin(t), 4, \sqrt{2}\cos(t) \rangle$

$\vec{r}\,'' = \langle -\sqrt{2}\cos(t), 0, -\sqrt{2}\sin(t) \rangle$

Need $t = \pi/2 \ldots$

$\vec{r}\,' = \langle -\sqrt{2}, 4, 0 \rangle$ $\quad |\vec{r}\,'| = \sqrt{18} = 3\sqrt{2}$

$\vec{r}\,'' = \langle 0, 0, -\sqrt{2} \rangle$ $\quad \boxed{K = \dfrac{6}{(3\sqrt{2})^3}}$

$\vec{r}\,' \times \vec{r}\,'' = \langle -4\sqrt{2}, -2, 0 \rangle$

$|\vec{r}\,' \times \vec{r}\,''| = \sqrt{32+4} = \sqrt{36} = 6$

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \left\langle -\frac{1}{3}\sin(t), \frac{2\sqrt{2}}{3}, \frac{1}{3}\cos(t) \right\rangle$$

$$\vec{T}\,' = \left\langle -\frac{1}{3}\cos(t), 0, -\frac{1}{3}\sin(t) \right\rangle$$

$$\vec{N} = \langle -\cos(t), 0, -\sin(t) \rangle$$

$$\vec{T}\left(\frac{\pi}{2}\right) = \left\langle -\frac{1}{3}, \frac{2\sqrt{2}}{3}, 0 \right\rangle$$

$$\vec{N}\left(\frac{\pi}{2}\right) = \langle 0, 0, -1 \rangle$$

$$\vec{B} = \left\langle -\frac{2\sqrt{2}}{3}, -\frac{1}{3}, 0 \right\rangle.$$

#6 $\vec{F} = m\vec{a}$

$$\vec{a} = \frac{1}{m}\vec{F} = \frac{1}{4}\langle t, t^2, 1\rangle$$

$$\int \vec{a} = \vec{v} = \frac{1}{4}\langle \tfrac{1}{2}t^2, \tfrac{1}{3}t^3, t\rangle + \vec{C}_1$$

$$\vec{v}(0) = \vec{C}_1 = \langle \tfrac{1}{2}, 1, \tfrac{1}{2}\rangle$$

$$\vec{r} = \int \vec{v} = \frac{1}{4}\langle \tfrac{1}{6}t^3, \tfrac{1}{12}t^4, \tfrac{1}{2}t^2\rangle + t\langle \tfrac{1}{2}, 1, \tfrac{1}{2}\rangle + \vec{r}(0)$$

$$d = |\vec{r}(2) - \vec{r}(0)|$$

$$d = \left| \frac{1}{4}\left\langle \frac{8}{6}, \frac{16}{12}, \frac{4}{2} \right\rangle + \langle 1, 2, 1 \rangle \right|$$

$$= \left| \left\langle \frac{1}{3}+1, \frac{1}{3}+2, \frac{1}{2}+1 \right\rangle \right|$$

$$= \left| \left\langle 4/3, 7/3, 3/2 \right\rangle \right|$$

$$d = \frac{\sqrt{341}}{6}.$$