Math 1060Q Lecture 7

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We shall discuss how to add, subtract, multiply and divide two functions and start thinking about the resulting graphs

- Some function notation
- Domain of new function
- Graphing sums and differences
- Example of graphing a product
- Graphing the reciprocal

Here is the notation for the four operations combining two functions.

- $f(x) + g(x) = (f + g)(x)$
- $f(x) - g(x) = (f - g)(x)$
- $f(x) \cdot g(x) = (f \cdot g)(x)$
- $f(x)/g(x) = (f/g)(x)$

Sometimes when the meaning of functions $f(x)$ and $g(x)$ is clear we drop the *argument* notationally, so the following may be encountered:

$$f + g \quad f - g \quad f \cdot g \quad \frac{f}{g}$$

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Generally, the domain of the new function will be the intersection of the domains of f and g, with one exception.

Let $\mathcal{D}_f$ be the domain of $f(x)$ and $\mathcal{D}_g$ be the domain of $g(x)$. We have the following:

- The domain of $f + g$ is $\mathcal{D}_f \cap \mathcal{D}_g$.
- The domain of $f - g$ is $\mathcal{D}_f \cap \mathcal{D}_g$.
- The domain of $f \cdot g$ is $\mathcal{D}_f \cap \mathcal{D}_g$.
- The domain of f/g is

$$\{x \text{ in } \mathcal{D}_f \cap \mathcal{D}_g \mid g(x) \neq 0\}.$$

So x will be in the domain only if it is already in both original domains $\mathcal{D}_f$ and $\mathcal{D}_g$... then just remember you also can't divide by zero.

Examples...

Example L7.1: Let $f(x) = x^2 + 6x - 4$ and $g(x) = 9 - 6x^2$. Find the new functions $f + g$, $f - g$, $f \cdot g$ and f/g along with their domains.
Solution: we have

$$f + g = -5x^2 + 6x + 5, \qquad f - g = 7x^2 + 6x - 13,$$
$$f \cdot g = (x^2 + 6x - 4)(9 - 6x^2), \qquad \frac{f}{g} = \frac{x^2 + 6x - 4}{9 - 6x^2}.$$

The domains are just $\mathbb{R}$, except in case of f/g. There, we must remove anywhere $g(x) = 0$:

$$9 - 6x^2 = 0 \Rightarrow x^2 = \frac{9}{6} = \frac{3}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}}.$$

So the domain for f/g is $\{x \mid x \neq \pm\sqrt{3/2}\}$.

Examples...

Example L7.2: Let $f(x) = x^2 - 3x + 2$ and $g(x) = \sqrt{x + 12}$. Find the domains of $f \pm g$, $f \cdot g$ and f/g.
Solution: Note that $\mathcal{D}_f = \mathbb{R}$ and $\mathcal{D}_g = [-12, \infty)$. In the case of $f \pm g$ and $f \cdot g$ it follows that the domain is

$$\mathcal{D}_f \cap \mathcal{D}_g = [-12, \infty).$$

A modification is needed in the case of f/g, since $g(-12) = 0$, so then the domain is

$$(-12, \infty).$$

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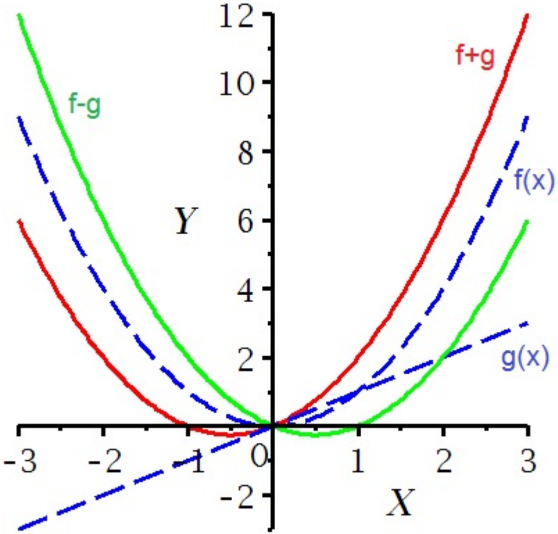
Given x, $f \pm g$ is found by adding or subtracting y-values



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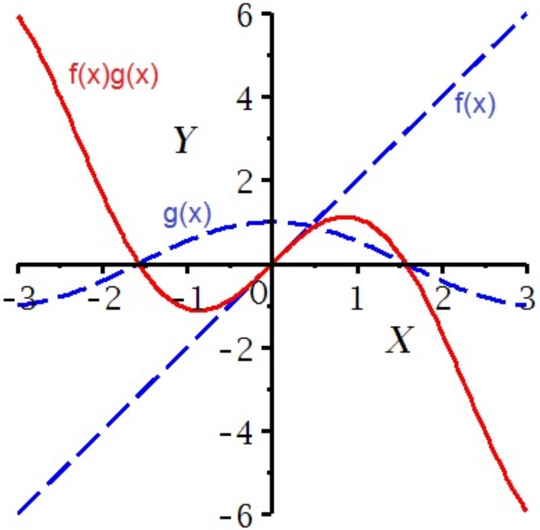
You can think of one function as a vertical stretching factor for the other; if the factor is negative, you also get a "flip"



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The reciprocal of $f(x)$ is $g(x) = 1/f(x)$.

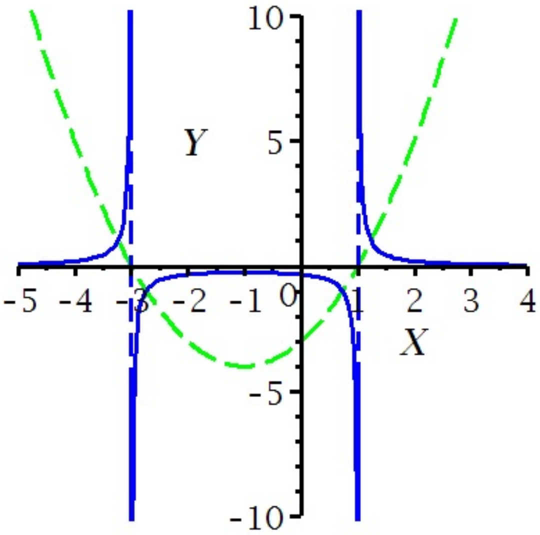
Some guidelines to graph the reciprocal:

- Wherever $f \to \pm\infty$, we have $g \to 0$.
- Note $f = g$ whenever $f = \pm 1$.
- f and g always have the same *sign*.
- f is very big when g is very small and vice-versa.
- g is undefined where $f = 0$; at these points we get vertical asymptotes.

Example L7.3: Sketch the inverse of $f(x) = x^2 + 2x - 3$.

- This parabola opens "up" and goes to ∞ as $x \to \pm\infty$. Thus the reciprocal goes to zero as $x \to \pm\infty$.
- Graphs of f and $1/f$ will cross at heights $y = \pm 1$.
- Note $f = 0 = x^2 + 2x - 3 = (x+3)(x-1)$ for $x = -3$ and $x = 1$. We have vertical asymptotes at these positions.
- Since f is very small near the asymptototes, the graph of $1/f$ "blows up" and follows the asymptotes vertically.

Example L7.3: Sketch the inverse of $f(x) = x^2 + 2x - 3$.



Practice!

Problem L7.1: Find the domains of the functions $f \cdot g$ and f/g, where $f(x) = -3x^2$ and $g(x) = \sqrt{x+1}$.

Problem L7.2: Find the domains of the functions $f + g$ and f/g, where $f(x) = \sqrt{1-x}$ and $g(x) = 4x^2 + 4x - 3$.

Problem L7.3: Sketch the graph of the reciprocal of f if $f(x) = -(x-2)^2 + 4$.