Math 1060Q Lecture 4

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Today we discuss more on functions

- Identifying the domain and range
- Odd and even functions
- Methods for graphing functions
- Equations for lines
- Parallel and perpendicular lines

The domain of $f(x)$ means what inputs x are allowed. The range means all the resulting values for f.

- $\mathcal{D}$: the domain.
- $\mathcal{R}$: the range.

Example L3.1: Find the domain and range of $f(x) = \frac{1}{x}$.
Solution: We can perform the operation $1/x$ for any number x except $x = 0$. Therefore,

$$\mathcal{D} = \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty).$$

To find the range, ask for which y there is some $x \neq 0$ and $y = 1/x$. Then try to find x in terms of y: $x = 1/y$ works if $y \neq 0$. Therefore,

$$\mathcal{R} = \{y \mid y \neq 0\} = (-\infty, 0) \cup (0, \infty).$$

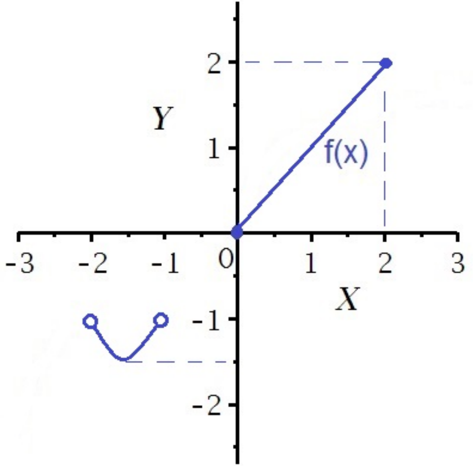
More examples would probably help...

Example L3.2: Find the domain and range of $f(x) = x^2$.
Solution: Is there any value x that we cannot multiply by itself to get x^2? (No) Therefore, $\mathcal{D} = \mathbb{R}$. However, we know $y = x^2 \geq 0$ is always true, thus $\mathcal{R} = [0, \infty)$.

Example L3.3: Find the domain and range of $f(x) = 5x$.
Solution: Is there any value x that we cannot multiply by 5? (No) Therefore, $\mathcal{D} = \mathbb{R}$. Also, we may always find x such that $y = 5x$ by simply taking $x = y/5$. Therefore, $\mathcal{R} = \mathbb{R}$.

We can read domains and ranges off of graphs too.

In this graph, we have $\mathcal{D} = (-2, -1) \cup [0, 2]$ and $\mathcal{R} = [-3/2, -1) \cup [0, 2]$.



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Some special cases are "odd" and "even" functions.

Definition (Odd function)

A function $f(x)$ is odd if $f(-x) = -f(x)$ for all x.

Definition (Even function)

A function $f(x)$ is even if $f(-x) = f(x)$ for all x.

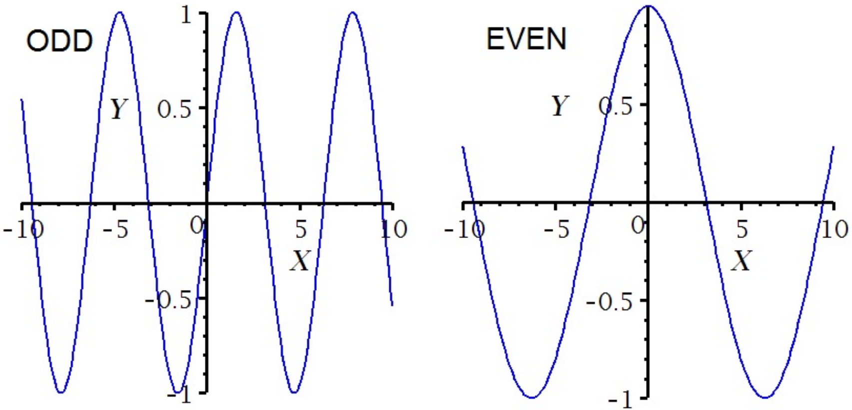
Example L3.4: Show that $f(x) = 2x$ is an odd function.
Solution: Just check what happens when we insert $-x$ instead of x:

$$f(-x) = 2(-x) = -2x = -(2x) = -f(x).$$

Example L3.5: Show that $f(x) = x^2 + 1$ is an even function.
Solution: Just check what happens when we insert $-x$ instead of x:

$$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x).$$

It is very easy to spot odd or even functions if you have their graphs.



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You need to set up your axes first, but how will you choose scaling and ranges for tick marks?

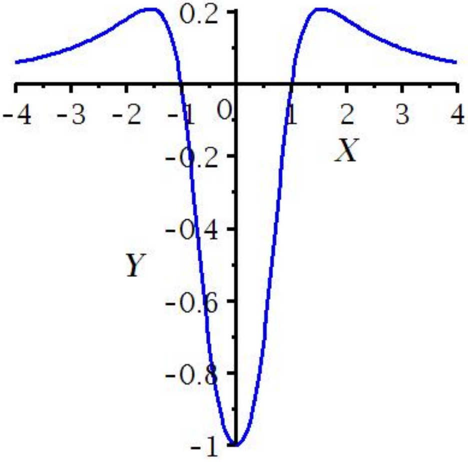
It helps to first collect the following information:

1. Domain and range
2. x-intercepts and y-intercepts
3. Local maximum and minimum points
4. Also any symmetry, for the shape of the graph.

Consider the graph of $f(x) = \frac{x^2-1}{x^4+1}$. In this case, the following hold:

- $\mathcal{D} = \mathbb{R}$, $\mathcal{R} \approx (-1, 0.21)$.
- y-intercept at $(0, -1)$ and x-intercepts at $(\pm 1, 0)$.
- Local minimum at $(0, -1)$.
- Local maximum points at around $(\pm 0.55, 0.21)$.
- y-axis symmetry.

Here is the graph



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We frequently encounter "linear" relationships between two quantities.

A linear relationship exists if a change in one quantity is related to a change in a other quantity by a constant amount.

- Force is mass multiplied by acceleration; $F = ma$.
- Tax T is the tax rate r multiplied by income I less credits c;

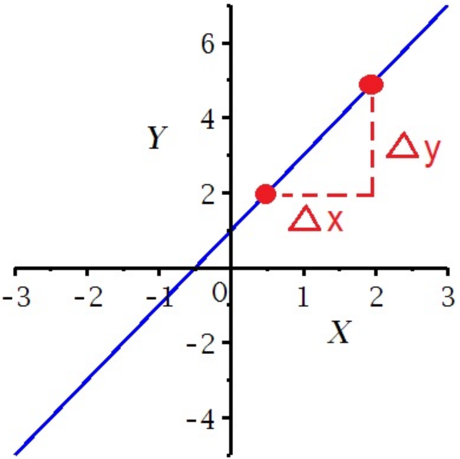
$$T = r(I - c).$$

We will now let x and y be the two quantities and explore two main expressions of a linear relationship:

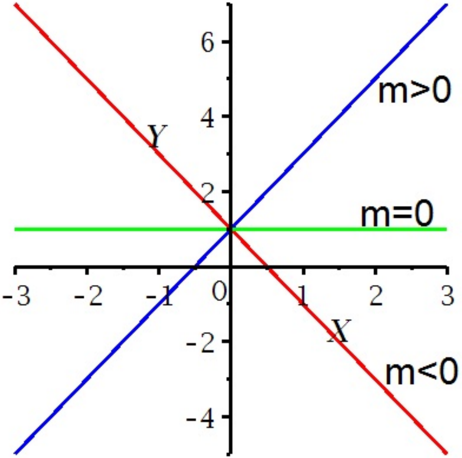
$$y = mx + b \quad \text{(slope-intercept form)}$$
$$y - y_0 = m(x - x_0) \quad \text{(point-slope form)}$$

The slope is m and measures how fast y can change relative to a change in x:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

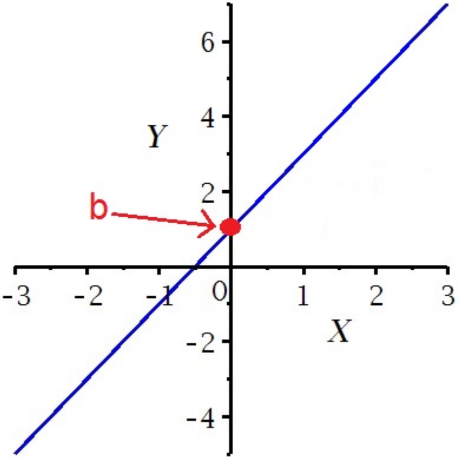


Lines with positive slope go “up” from left to right; those with negative slope go down



Slope-intercept form

The y-intercept is $(0, b)$.



So one calculates m and b, then plugs them into the formula

$$y = mx + b.$$

Some examples.

Example L3.6: What are the slope and y-intercept of the line given by

$$2y = 4x + 10.$$

Solution: Put into slope-intercept form by dividing through by 2:

$$y = 2x + 5.$$

The slope is $m = 2$ and the y-intercept is $(0, 5)$.

Example L3.7: If a line has slope -3 and y-intercept $(0, 1)$, what is an equation for the line?
Solution: $m = -3$ and $b = 1$, so $y = -3x + 1$.

Point-slope form is better if you want to know what line passes through two arbitrary points.

Consider the line passing through (x_1, y_1) and (x_2, y_2). Then the slope of the line is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

One you have this, it is not immediately clear what the y-intercept is, so you cannot yet use slope-intercept form. The quick solution is to pick one of the two points, call this (x_0, y_0), and plug into the point-slope formula:

$$y - y_0 = m(x - x_0).$$

In other words, either write

$$y - y_1 = m(x - x_1) \;\text{ or else }\; y - y_2 = m(x - x_2).$$

These two equations are equivalent.

Examples...

Example L3.8: Find the equation of the line passing through $(3, -1)$ and $(-1, 2)$.
Solution: The slope is

$$m = \frac{2 - (-1)}{-1 - 3} = \frac{3}{-4} = -\frac{3}{4}.$$

Choose $(x_0, y_0) = (3, -1)$ and apply point-slope:

$$y - (-1) = y + 1 = -\frac{3}{4}(x - 3).$$

Example L3.9: Find the equation of the line passing through $(-2, 5)$ with slope $m = 17$.
Solution: The slope and a point that the line passes through are both given, so we simply apply the point-slope formula to get

$$y - 5 = 17(x + 2).$$

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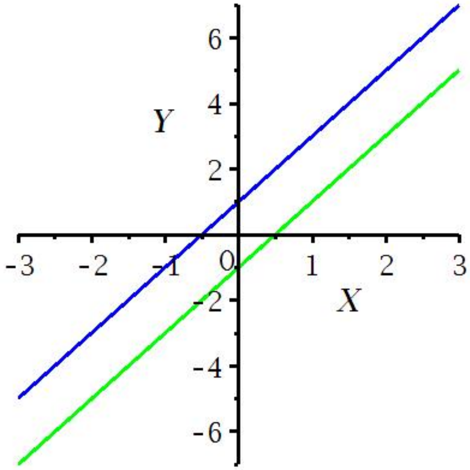
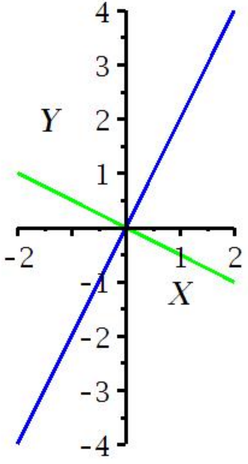
Parallel lines have the same slope



Example: $y = 2x + 1$ and $y = 2x - 1$.

Perpendicular lines lie at right angles to each other. If one has slope m, the other has slope $-1/m$.

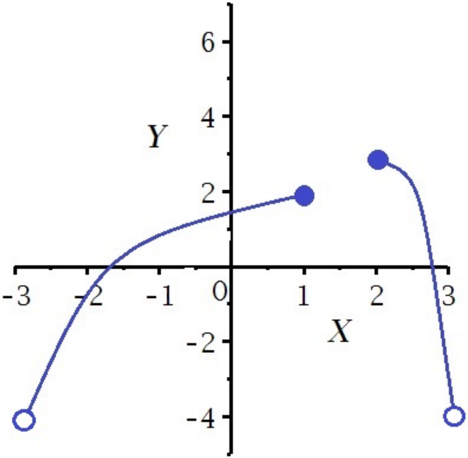


Example: $y = 2x$ and $y = -\frac{1}{2}x$.

Practice! More on next slide...

Problem L4.1: What is the domain of $f(x) = \frac{x-1}{x+1}$?

Problem L4.2: What are the domain and range of the function?



Practice!

Problem L4.3: Is the function $f(x) = x^3$ even? odd? What about $f(x) = x^4$?

Problem L4.4: Find the equation of the line with slope 4 and y-intercept at $(0, 8)$.

Problem L4.5: Find the line passing through $(2, 3)$ and perpendicular to $y = -6x + 4$.

Problem L4.6: Find the line passing through $(1, 2)$ and parallel $y = -6x + 4$.