Math 1060Q Lecture 3

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Today we discuss equations, graphs and functions

- Conditional equations
- x and y intercepts
- Symmetry
- Definition: what is a function?
- Vertical line test

Some equations always hold (identities), others only hold for certain values of x (conditional)

Here is an identity:

$$\frac{(x-1)(x^2+1)}{x^2+1} = x - 1.$$

This equation is true for every real number x. An example of a conditional equation would be:

$$x^2 - 5x = -6.$$

In order to find for which values x this holds, proceed as follows:

$$\begin{aligned} x^2 - 5x + 6 = 0 \;&\Rightarrow\; (x-3)(x-2) = 0 \\ &\Rightarrow\; x = 3 \text{ or } x = 2. \end{aligned}$$

We will study methods to solve certain equations as the semester progresses.

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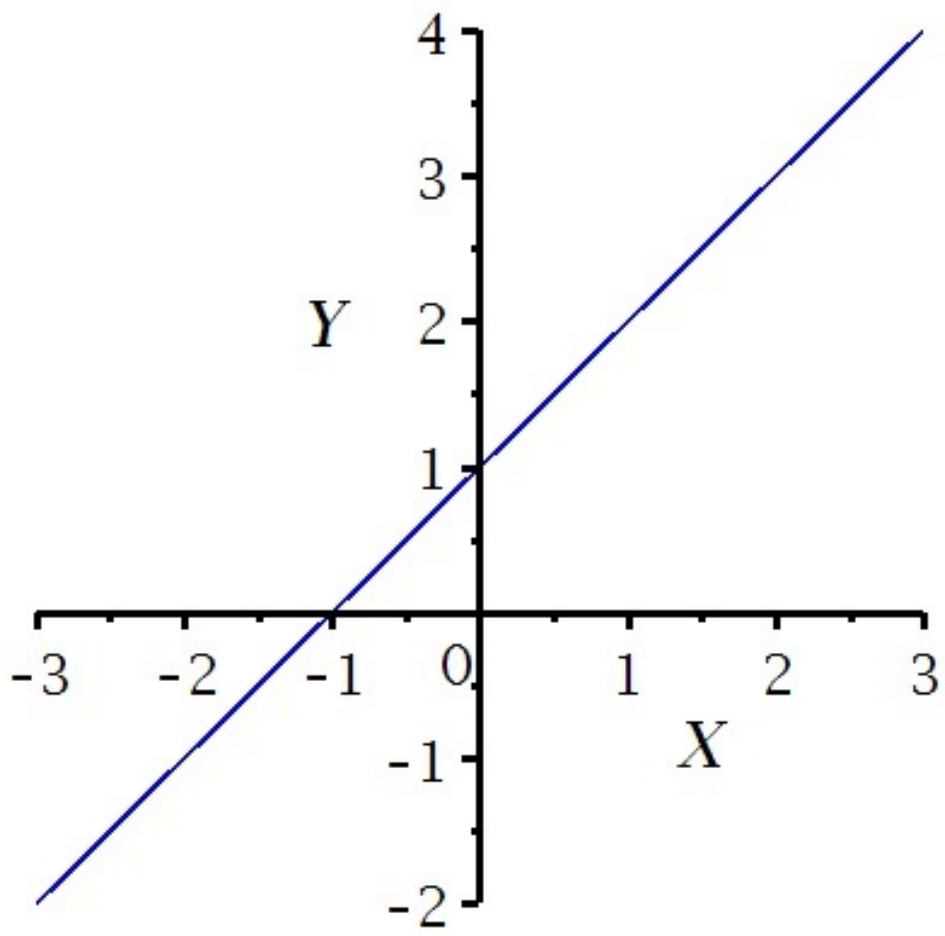
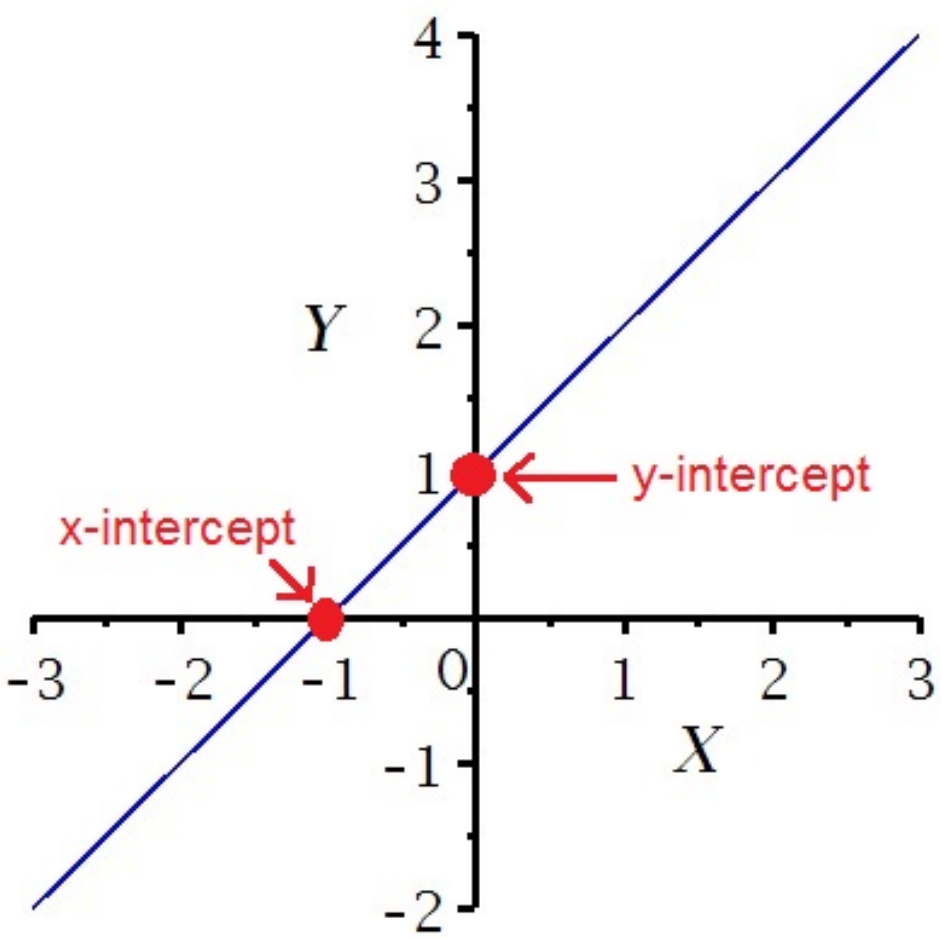
It is often useful to graph equations

Consider an equation with both x and y, such as $y = x + 1$. We graph this by marking all points in the xy-plane that satisfy the equation:



You will want to be able to identify x and y intercepts

An x-intercept is anywhere the graph crosses the x-axis. Similary, a y-intercept is anywhere the graph crosses the y-axis.



Example L3.1: Find the x and y intercepts for the graph of the equation $2y = 5x - 3$.
Solution: To find the x-intercept, we set $y = 0$ and see that

$$0 = 5x - 3 \;\Rightarrow\; x = \frac{3}{5}.$$

The x-intercept is at the point $(3/5, 0)$ on the graph. To find the y-intercept, set $x = 0$ and solve for y:

$$2y = -3 \;\Rightarrow\; y = -3/2.$$

The y-intercept is at the point $(0, -3/2)$.

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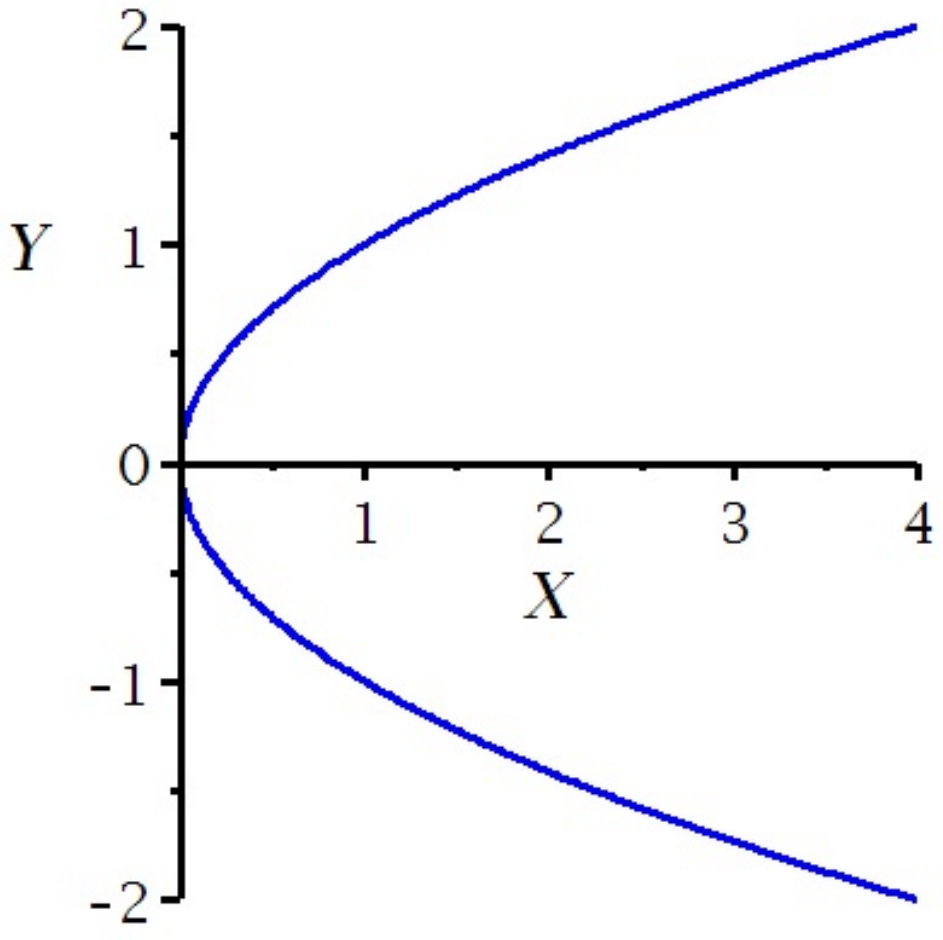
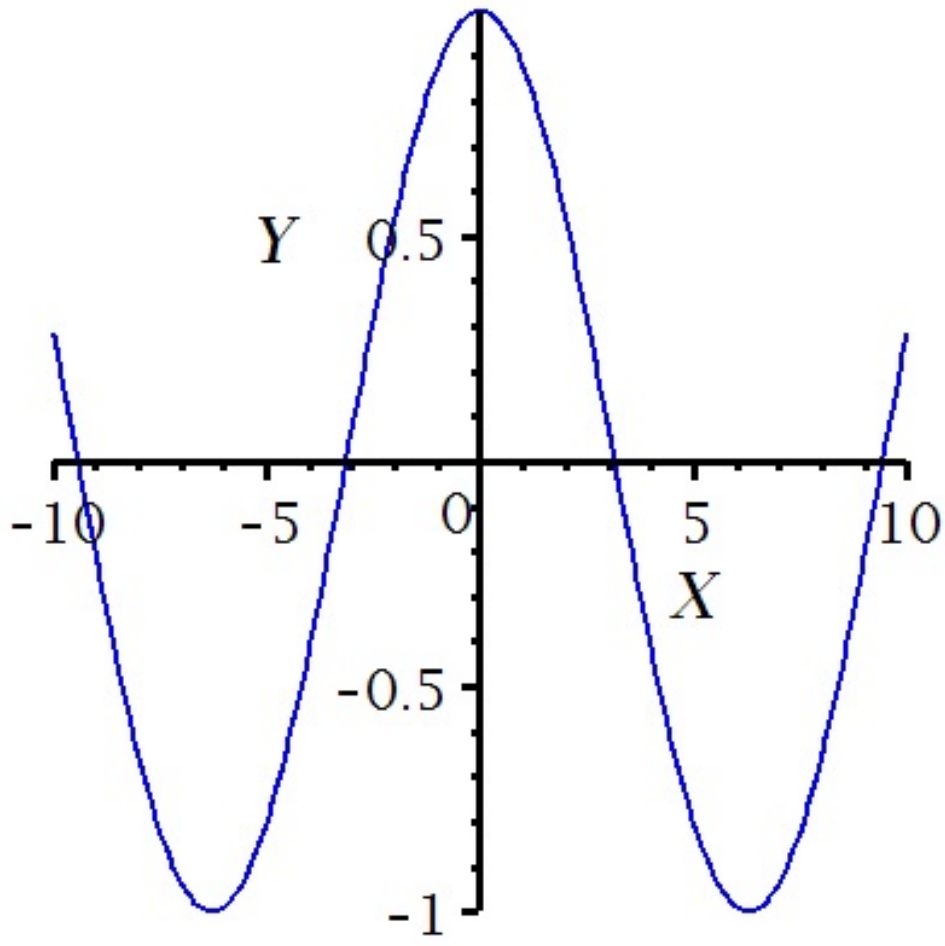
Three common types of symmetry found in graphs are (1) x-axis symmetry, (2) y-axis symmetry and (3) origin symmetry

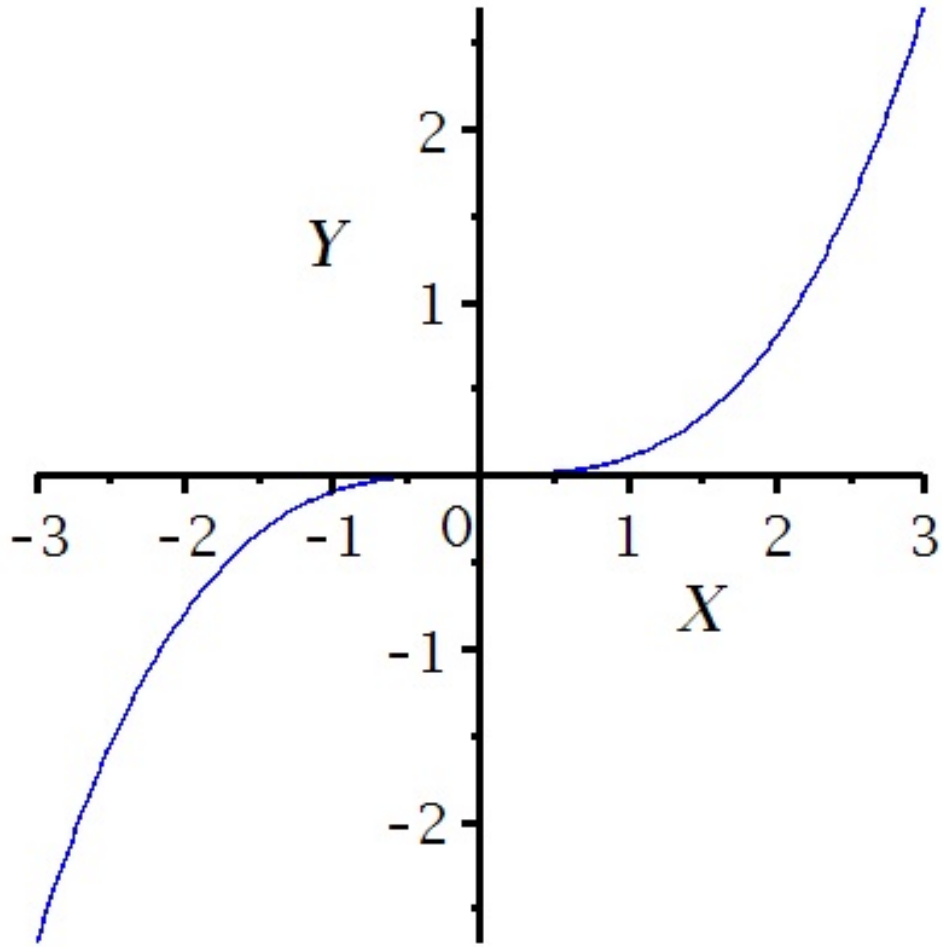
x-axis symmetry just means the graph looks like it is mirrored across the x-axis, e.g.



y-axis symmetry just means the graph looks like it is mirrored across the y-axis



Origin symmetry means the graph looks the same if it is rotated by 180 degrees about the origin



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A function takes in a number, performs some operation, and outputs the result

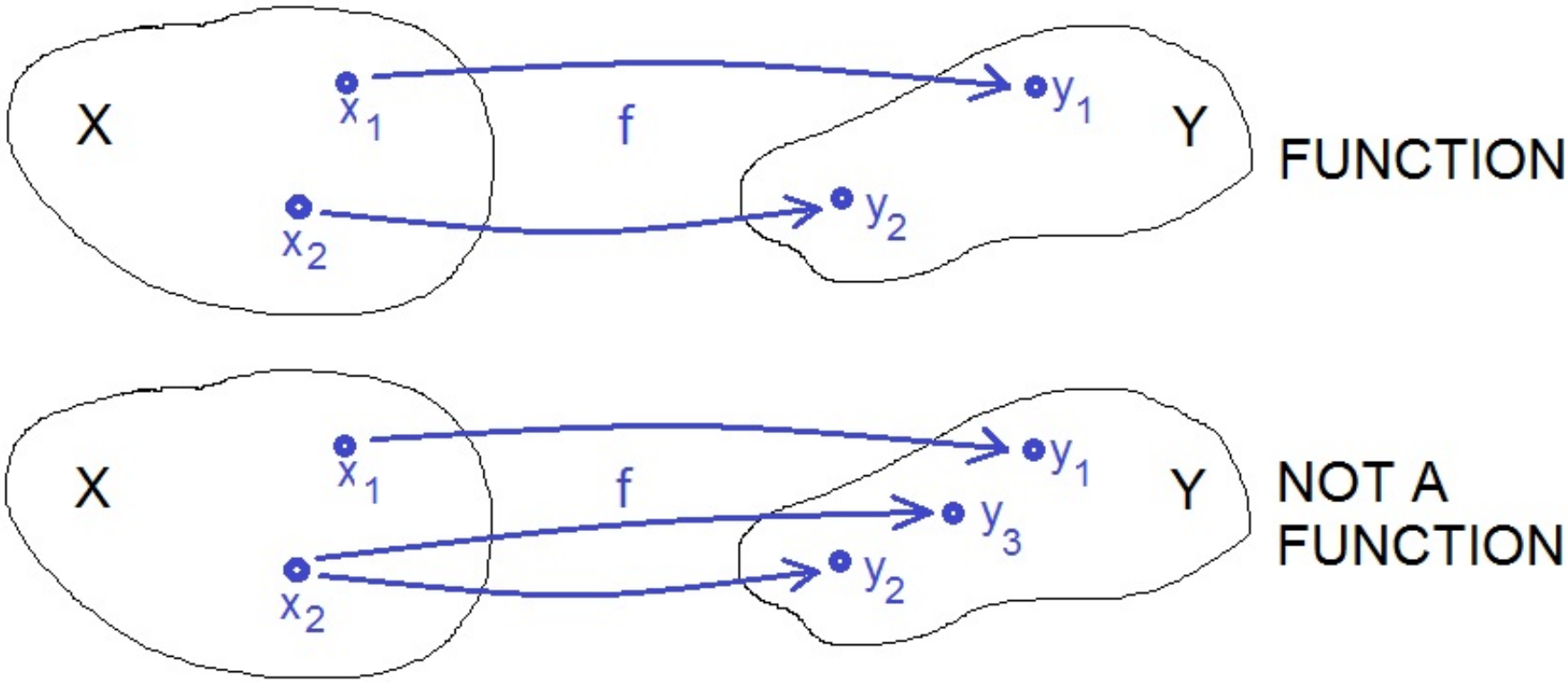
Definition (Function)

A function from a set X to a set Y is a rule that assigns each element in X to precisely one element in Y.

Consider that the volume V of a sphere is calculated in terms of its radius r as $V = \frac{4}{3}\pi r^3$. We say that $V = V(r)$, meaning V is a function of r.

$$V(1) = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$$
$$V(2) = \frac{4}{3}\pi(2)^3 = \frac{4}{3}\pi 8 = \frac{32}{3}\pi$$
$$V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi 27 = \frac{108}{3}\pi$$

If a rule assigns one number in X to more than one number in Y, it is not a function



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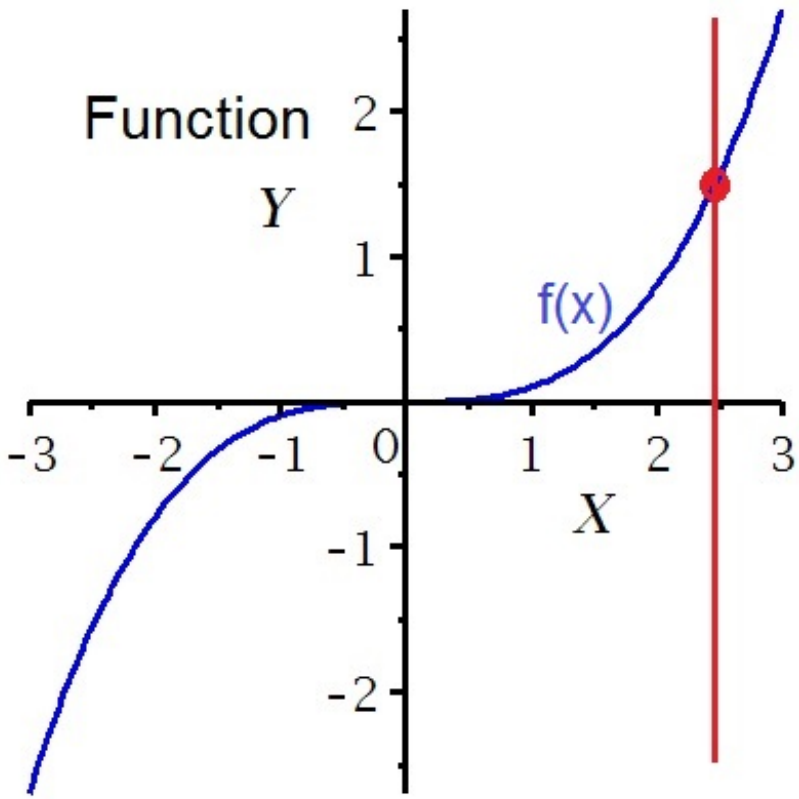
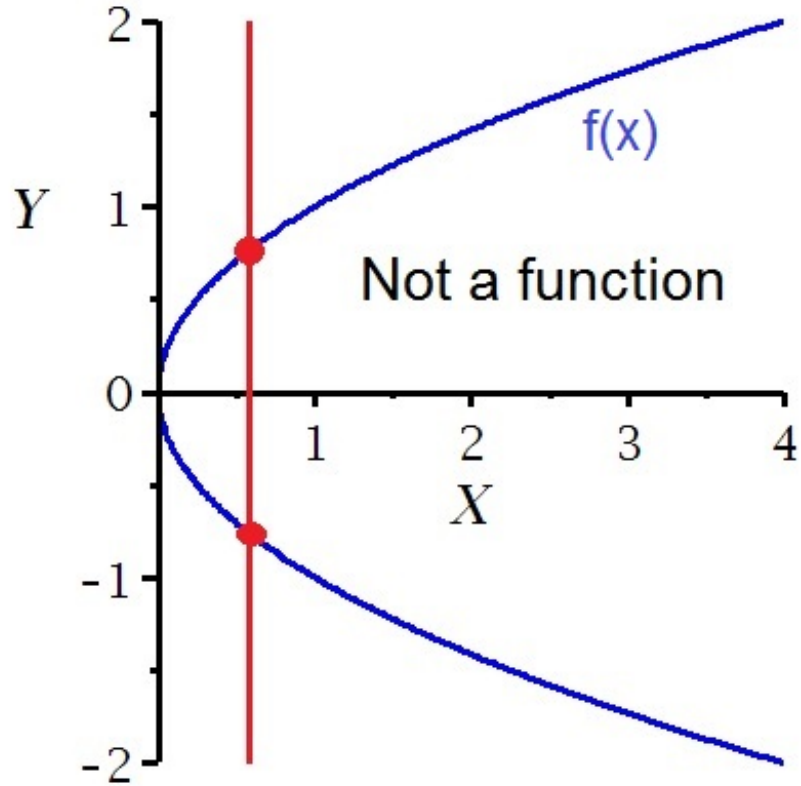
Given graphs as below, $f(x)$ is a function ONLY if an arbitrary vertical line intersects the graph exactly one time

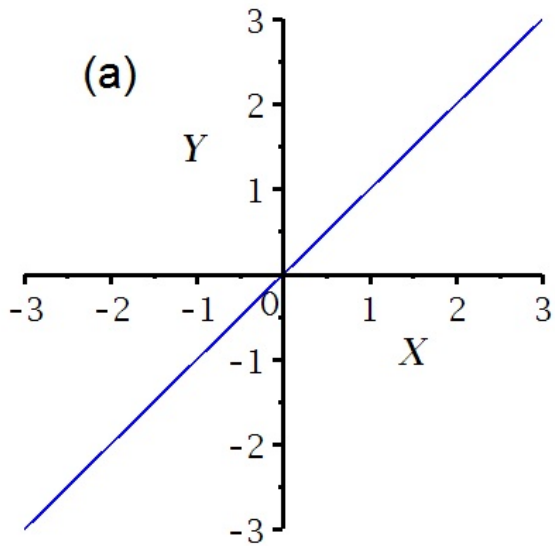


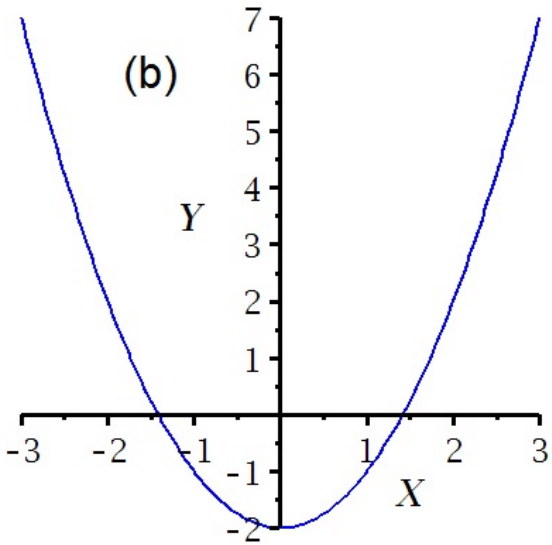


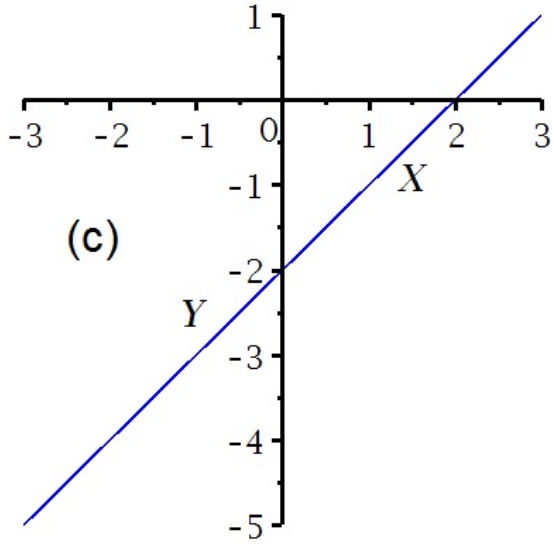
Practice problems! More on next slide...

Problem L3.1: Find any x or y intercepts for the graph of $-2y + 6x = 4$.

Problem L3.2: What kinds of symmetry do these graphs have (if any)?

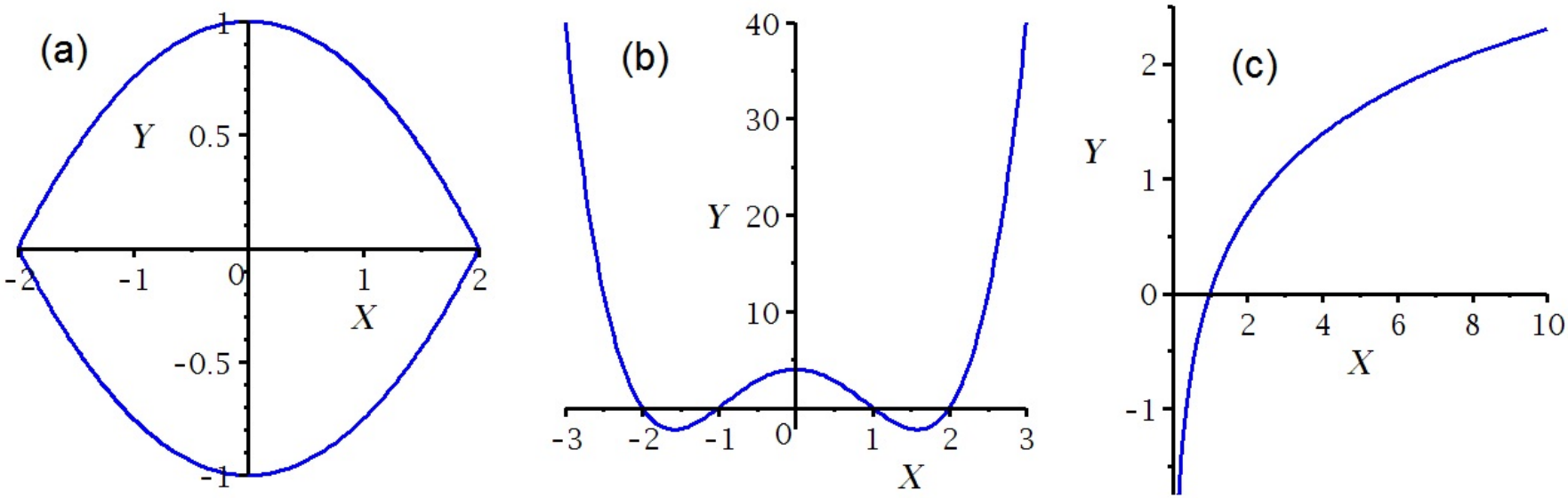






Practice problems!

Problem L3.3: Which of these are functions?



Problem L3.4: Find any x or y intercepts for the graph of $y = x^2 - 4$.