Math 1060Q Lecture 20

Jeffrey Connors

University of Connecticut

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Exponential functions are commonly used to model a wide variety of behaviors

- Review: properties of exponents.
- What is an exponential function?
- Graphing exponentials.
- The natural exponential function.

These properties help to work with exponential functions and with algebra in general.

Rule	Example
$a^{r_1+r_2} = a^{r_1}a^{r_2}$	$2^{3+4} = 2^3 2^4$
$a^{r_1-r_2} = a^{r_1}/a^{r_2}$	$3^{3-4} = 3^3/3^4$
$(a^{r_1})^{r_2} = a^{r_1 r_2}$	$(4^5)^6 = 4^{5\cdot 6} = 4^{30}$
$a_1^r a_2^r = (a_1 a_2)^r$	$2^3 2^4 = (2\cdot 3)^4 = 6^4$
$(a_1)^r/(a_2)^r = (a_1/a_2)^r$	$2^4/3^4 = (2/3)^4$

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The key is to recognize when x occurs as the *exponent*

- The function $y = 2^x$ is an *exponential function*.
- The function $y = x^2$ is NOT an exponential function.
- Generally, $y = c\left(a^{bx}\right) + d$ with a, b, c and d non-zero, real numbers is an exponential function.

Think about some values of this type of function:

x	$y = 2^x$
-3	$1/8$
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8

If the base a is a positive fraction, we see a different behavior.

Consider $0 < a < 1$, for example $y = (1/2)^x$. We may also write this as

$$y = \left(\frac{1}{2}\right)^x = \left(2^{-1}\right)^x = 2^{-x}.$$

Note that this is equivalent to $b = -1$ with $a = 2 > 1$. Here are some values of this function:

x	$y = 2^{-x}$
-3	8
-2	4
-1	2
0	1
1	$1/2$
2	$1/4$
3	$1/8$

- Review: properties of exponents.
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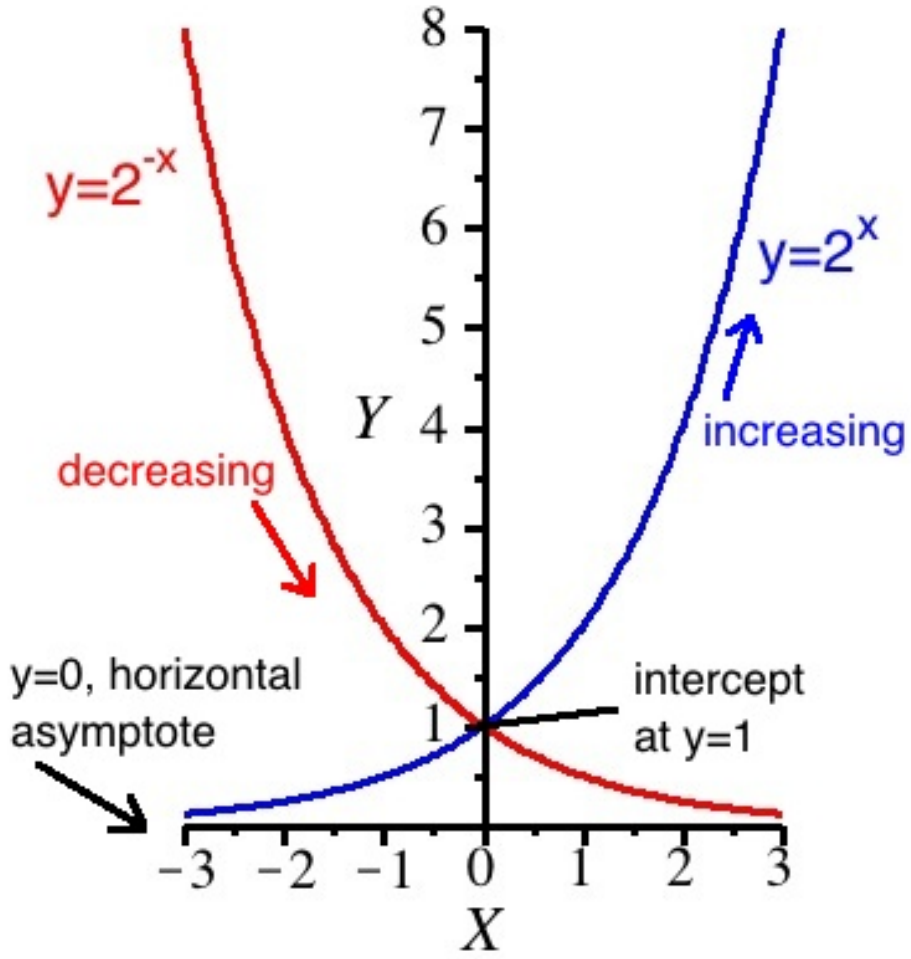
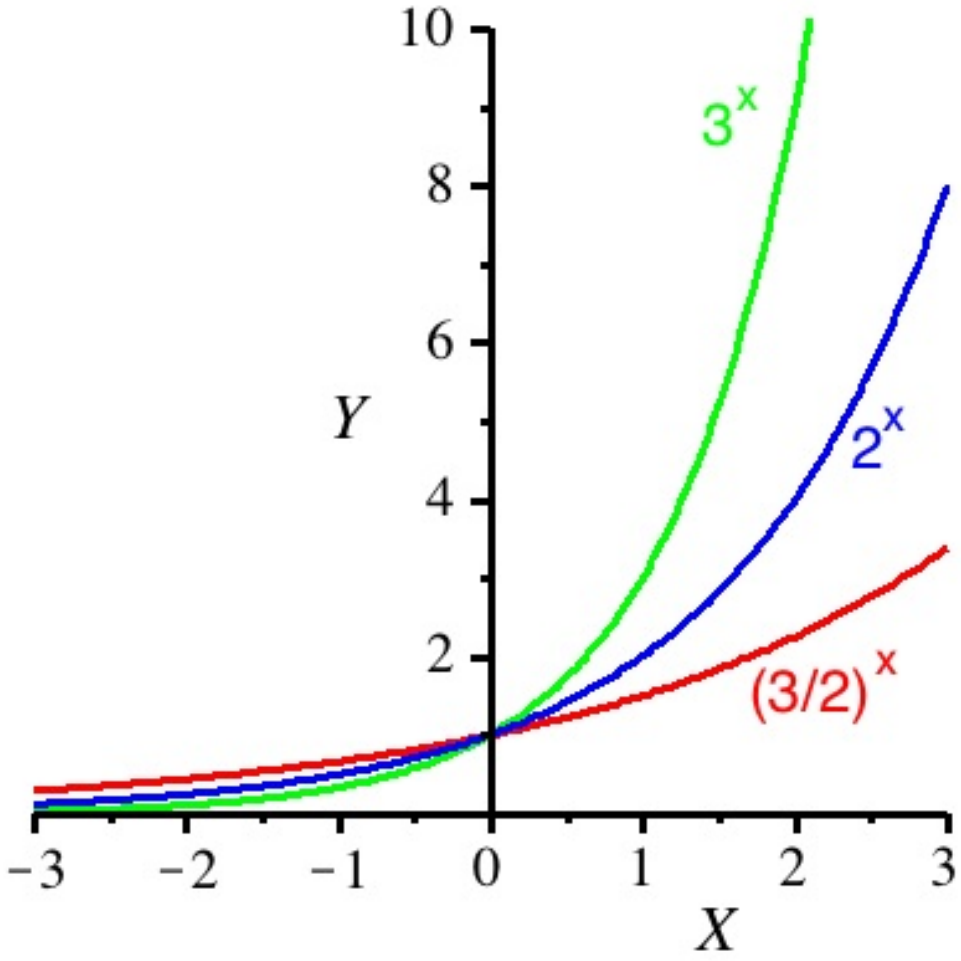
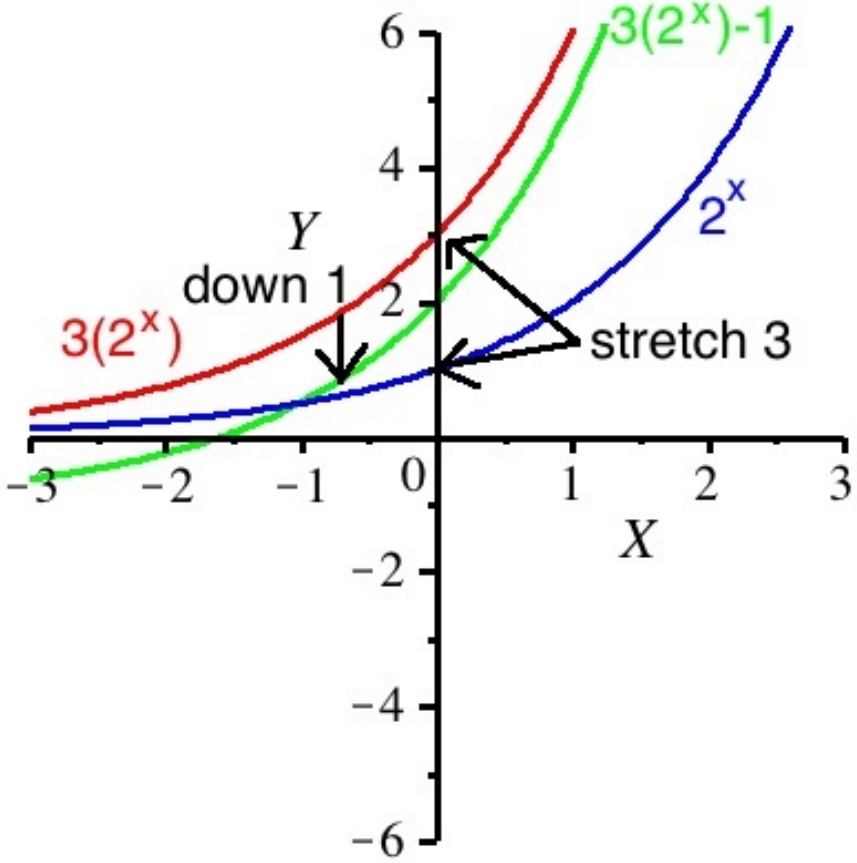
There are two main graphs to have in mind.

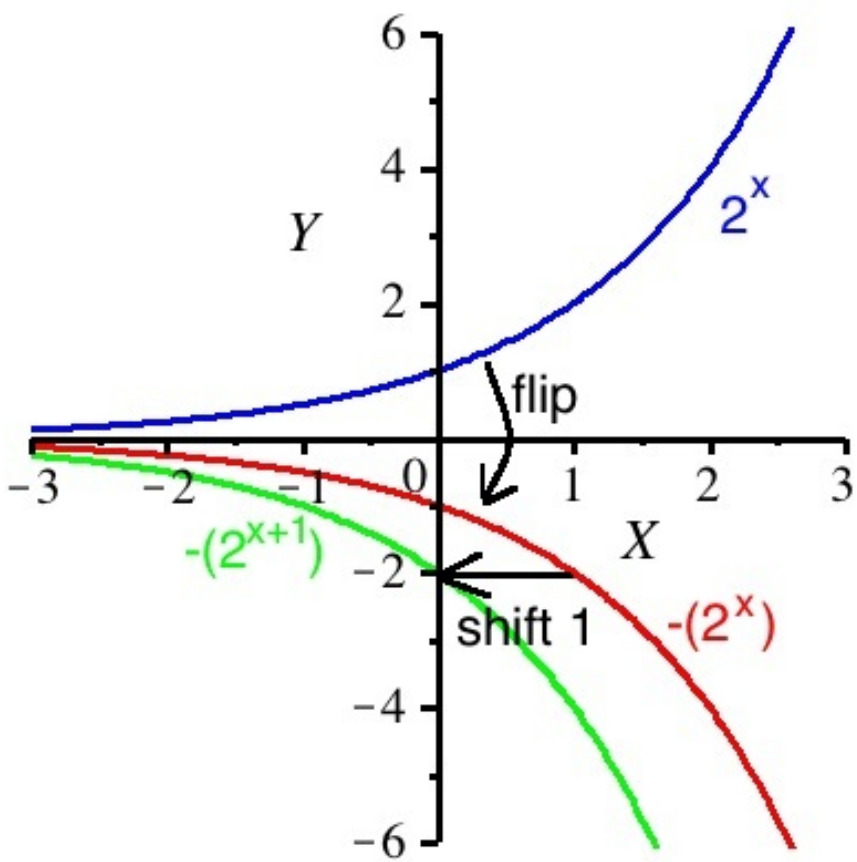


The function grows more quickly for larger base values a.



As usual, we can stretch and shift these functions.





- Review: properties of exponents.
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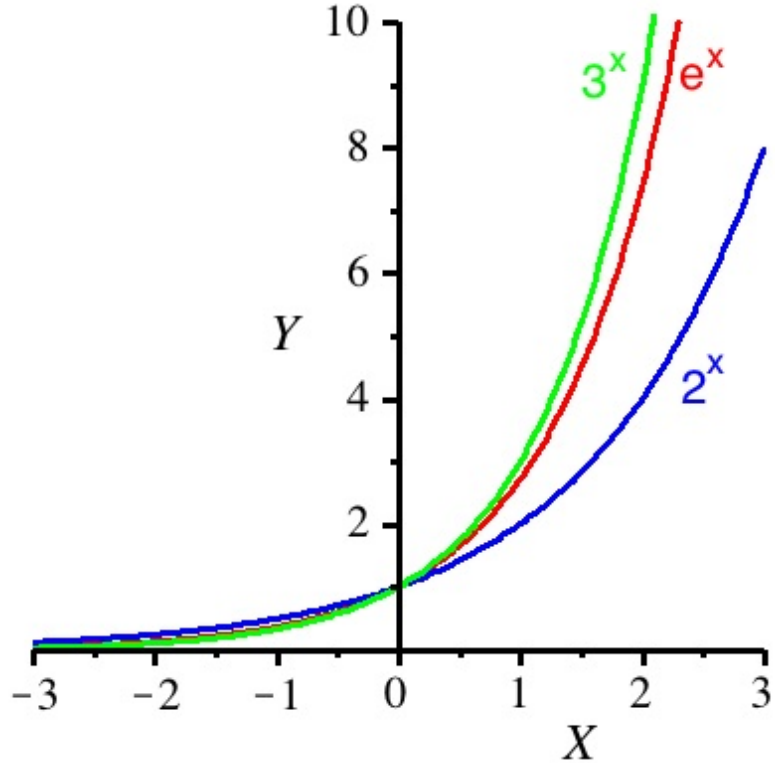
The natural exponential function occurs frequently in numerous applications.

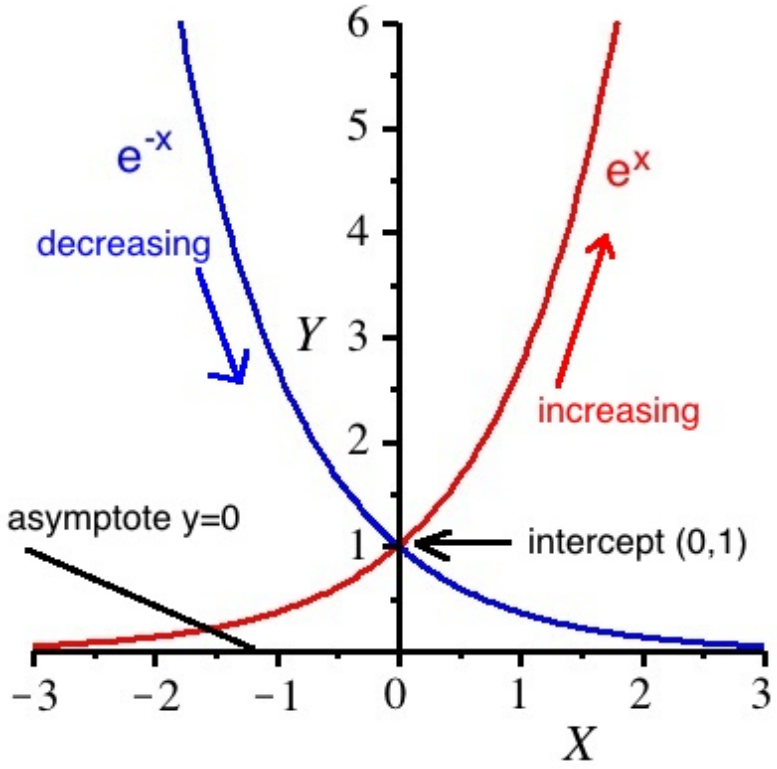
The *natural exponential function* is

$$f(x) = e^x, \quad e \approx 2.71828\ldots$$

is a special irrational number, like π. Note that $2 < e < 3$:



As with the other exponential functions, be sure you understand the graphs of both e^x and e^{-x}.



Algebraic properties.

Here are a few of the more common operations one encounters with e^x:

Rule	Example
$e^{x_1}e^{x_2} = e^{x_1+x_2}$	$e^2e^3 = e^{2+3} = e^5$
$\left(e^{x_1}\right)^{x_2} = e^{x_1x_2}$	$\left(e^4\right)^2 = e^{4\cdot 2} = e^8$
$e^{x_1}/e^{x_2} = e^{x_1-x_2}$	$e^5/e^4 = e^{5-4} = e$

Practice!

Problem L20.1: Sketch the graph of $y = 2e^{-x} + 1$.

Problem L20.2: Sketch the graph of $y = 4^x$.

Problem L20.3: Solve the equation $e^{2x} = e^6 e^{3x}$ for x.