Math 1060Q Lecture 19

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Inverse trigonometric functions

- The general purpose of these functions.
- Inverse of $\sin(x)$.
- Inverse of $\cos(x)$.
- Inverse of $\tan(x)$.

We seek to answer questions such as "if $\sin(x) = 1$, then what is x?"

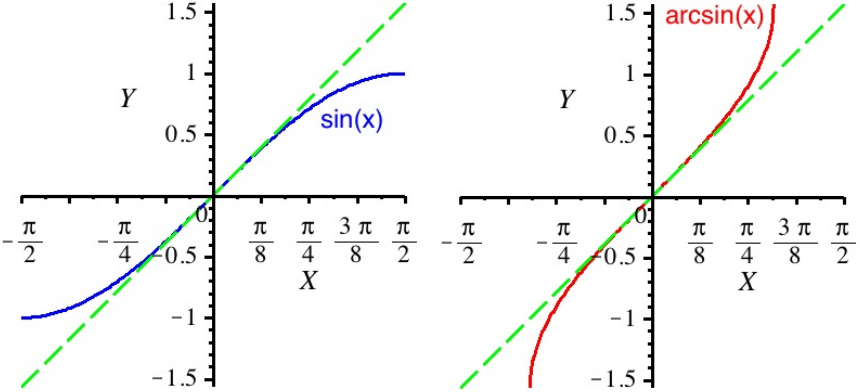
If $f(x) = \sin(x) = y$ then we know $x = f^{-1}(y)$, but what is f^{-1}?

- The inverse does not generally exist! Recall the graph of a sinusoid and apply the horizontal line test; these are not 1-to-1 functions.
- For example, if $f(x) = \sin(x) = 1$ then $x = \pi/2 + 2k\pi$; there are infinite solutions and $f^{-1}(1) = \pi/2 + 2k\pi$ is not well-defined.
- We get around this by *restricting* the domain for $\sin(x)$ when we discuss the inverse.
- Similarly, we can discuss inverses for $\cos(x)$ and $\tan(x)$.

- The general purpose of these functions.
- Inverse of $\sin(x)$.
- Inverse of $\cos(x)$.
- Inverse of $\tan(x)$.

Restrict the domain for $\sin(x)$ to $[-\pi/2, \pi/2]$.

We say $\sin(x) = y \iff x = \arcsin(y)$. Recall how we sketch the inverse:



Some details for $\arcsin(x)$.

- Another common notation is $\arcsin(x) = \sin^{-1}(x)$. This is not to be confused with

$$(\sin(x))^{-1} = \frac{1}{\sin(x)} = \csc(x).$$

- The domain of $\sin(x)$ is restricted to $[-\pi/2, \pi/2]$ but we achieve the full range $[-1, 1]$.
- The domain of $\arcsin(x)$ is $[-1, 1]$ and the range is $[-\pi/2, \pi/2]$.
- The relationship $\arcsin(\sin(x)) = x$ is not always true here. This is illustrated below.

Example L19.1: Find $\arcsin(\sin(\pi))$.

Solution: We know that $\sin(\pi) = 0$. However, the range of $\arcsin(x)$ is $[-\pi/2, \pi/2]$ and $\sin(0) = 0$, so $\arcsin(0) = 0$; *i.e.*

$$\arcsin(\sin(\pi)) = 0 \neq \pi.$$

When you figure out $\arcsin(x) = \theta$, look only at θ in the first and fourth quadrants.

Example L19.2: Find $\arcsin(1/\sqrt{2})$.

Solution: We note that $x = \arcsin(1/\sqrt{2}) \iff \sin(x) = 1/\sqrt{2}$. Recall that this is true for $x = \pi/4$ and also $x = 3\pi/4$. However, since only $x = \pi/4$ is in either the first or fourth quadrants, we say $\arcsin(1/\sqrt{2}) = \pi/4$.

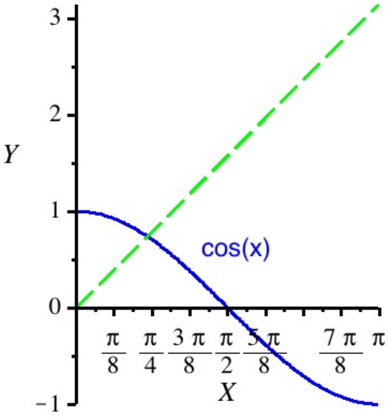
Example L19.3: Find $\arcsin(-1/2)$.

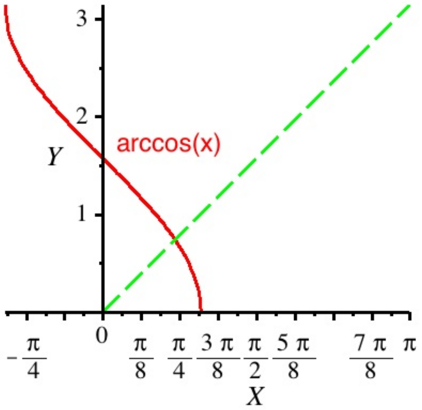
Solution: You might say that since we want $\sin(x) = -1/2$ then $x = 7\pi/6$ or $x = 11\pi/6$. These are both wrong, but $x = 11\pi/6$ corresponds to the negative angle $x = -\pi/6$, which is the correct answer; $x = \arcsin(-1/2) = -\pi/6$, since this is the only possibility with $-\pi/2 \leq x \leq \pi/2$.

- The general purpose of these functions.
- Inverse of $\sin(x)$.
- Inverse of $\cos(x)$.
- Inverse of $\tan(x)$.

To define arccos(x), we restrict the domain of cos(x) to $[0, \pi]$.

Sketch of arccos(x):





Some details for arccos(x).

- Another common notation is $\arccos(x) = \cos^{-1}(x)$.
- The domain of $\cos(x)$ is restricted to $[0, \pi]$ but we achieve the full range $[-1, 1]$.
- The domain of $\arccos(x)$ is $[-1, 1]$ and the range is $[0, \pi]$.
- As with $\sin(x)$, the relationship $\arccos(\cos(x)) = x$ is not always true here.

Example L19.4: Find $\arccos(\cos(3\pi/2))$.

Solution: We know that $\cos(3\pi/2) = 0$. The range of $\arccos(x)$ is $[0, \pi]$ and $\cos(\pi/2) = 0$, so $\arccos(0) = \pi/2$; *i.e.*

$$\arccos(\cos(3\pi/2)) = \pi/2 \neq 3\pi/2.$$

Some examples to calculate arccos(x).

Example L19.5: Calculate $\arccos(1/\sqrt{2})$.

Solution: We note that $y = \arccos(1/\sqrt{2}) \iff \cos(y) = 1/\sqrt{2}$. Recall this is true for $y = \pi/4$, which is in the interval $[0, \pi]$ and thus we say $\arccos(1/\sqrt{2}) = \pi/4$.

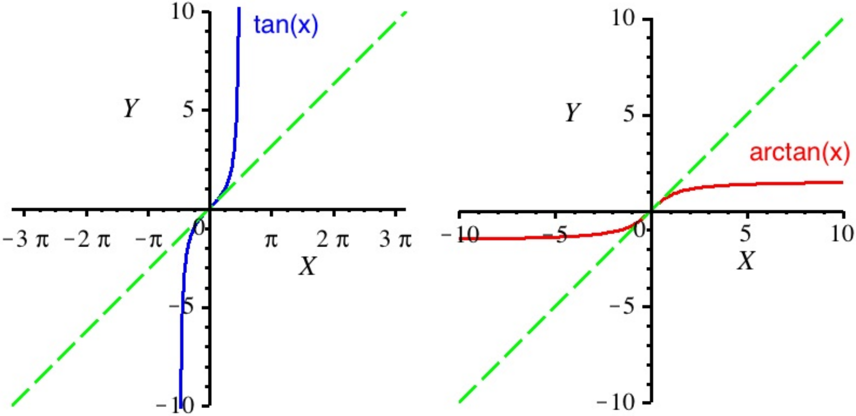
Example L19.6: Calculate $\arccos(-\sqrt{3}/2)$.

Solution: We need to identify $\cos(y) = -\sqrt{3}/2$ with $0 \leq y \leq \pi$. We look in the second quadrant; $\cos(5\pi/6) = -\sqrt{3}/2$, hence $\arccos(-\sqrt{3}/2) = 5\pi/6$.

- The general purpose of these functions.
- Inverse of $\sin(x)$.
- Inverse of $\cos(x)$.
- Inverse of $\tan(x)$.

The restriction of the domain for $\tan(x)$ is the same as for $\sin(x)$.

Sketch of $\arctan(x)$:



Some details for $\arctan(x)$.

- Another common notation is $\arctan(x) = \tan^{-1}(x)$.
- The domain of $\tan(x)$ is restricted to $[-\pi/2, \pi/2]$ but we achieve the full range $(-\infty, \infty)$.
- The domain of $\arctan(x)$ is $(-\infty, \infty)$ and the range is $[-\pi/2, \pi/2]$.
- The relationship $\arctan(\tan(x)) = x$ is not always true.

Example L19.7: Find $\arctan(\tan(\pi))$.

Solution: We know that $\tan(\pi) = 0$. However, $\tan(0) = 0$, and since the range of $\arctan(x)$ is $[-\pi/2, \pi/2]$ we must say $\arctan(0) = 0$, not $\arctan(0) = \pi$. Thus,

$$\arctan(\tan(\pi)) = 0 \neq \pi.$$

Examples to calculate arctan(x).

Example L19.8: Find arctan(1).

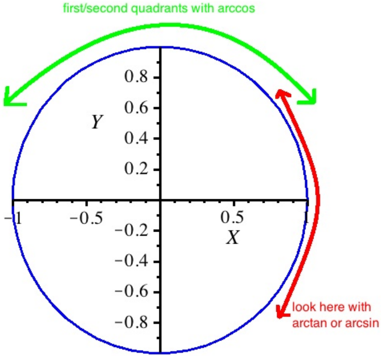
Solution: Recall that $\tan(\pi/4) = 1$. Since the angle $\pi/4$ is in the range $[-\pi/2, \pi/2]$ of arctan(x), we say that $\arctan(1) = \pi/4$.

Example L19.9: Find $\arctan(-\sqrt{3})$.

Solution: Recall that $\tan(-\pi/3) = -\sqrt{3}$. Since the angle $-\pi/3$ is between $-\pi/2$ and $\pi/2$, it follows that $\arctan(-\sqrt{3}) = -\pi/3$. Note that while $\tan(2\pi/3) = -\sqrt{3}$ is also true, we do not say that $\arctan(-\sqrt{3}) = 2\pi/3$ because the angle $2\pi/3$ is not in the interval $[-\pi/2, \pi/2]$.

Visualization of where to look for angles to get arcsin, arccos, arctan.

Where to look for θ if solving $\arcsin(x) = \theta$, $\arccos(x) = \theta$, or $\arctan(x) = \theta$.



Practice!

Problem L19.1: Fill in the table...

x	$1/\sqrt{2}$	$-1/\sqrt{2}$	1	-2
$\arcsin(x)$				
$\arccos(x)$				

Problem L19.2: Fill in the table...

x	0	-1	$-1/\sqrt{3}$	$1/\sqrt{3}$
$\arctan(x)$				

Practice!

Problem L19.1: Fill in the table...

x	$1/\sqrt{2}$	$-1/\sqrt{2}$	1	-2
$\arcsin(x)$	$\pi/4$	$-\pi/4$	$\pi/2$	—
$\arccos(x)$	$\pi/4$	$3\pi/4$	0	—

Problem L19.2: Fill in the table...

x	0	-1	$-1/\sqrt{3}$	$1/\sqrt{3}$
$\arctan(x)$	0	$-\pi/4$	$-\pi/6$	$\pi/6$