Math 1060Q Lecture 14

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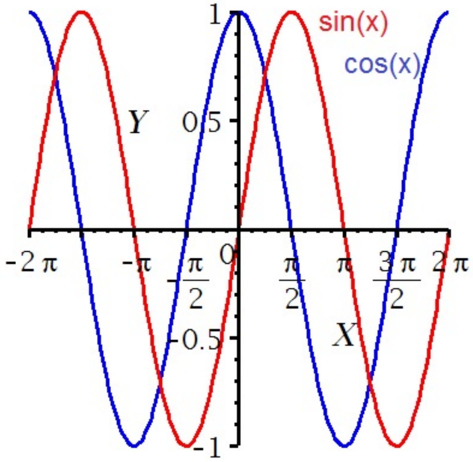
Today's goal is to become more familiar with sinusoidal graphs

- Graphs of $\sin(x)$ and $\cos(x)$.
- Controlling amplitude, period and shift.

You will want to know the graphs of $\sin(x)$ and $\cos(x)$.



- Amplitude (height) is 1.
- Period is $\tau = 2\pi$.
- Notice $\sin(x + \pi/2) = \cos(x)$.

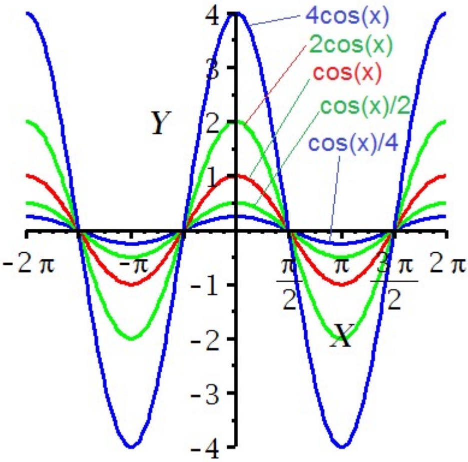
- Graphs of $\sin(x)$ and $\cos(x)$.
- Controlling amplitude, period and shift.

We want to understand $A\cos(Bx + C)$ and $A\sin(Bx + C)$.

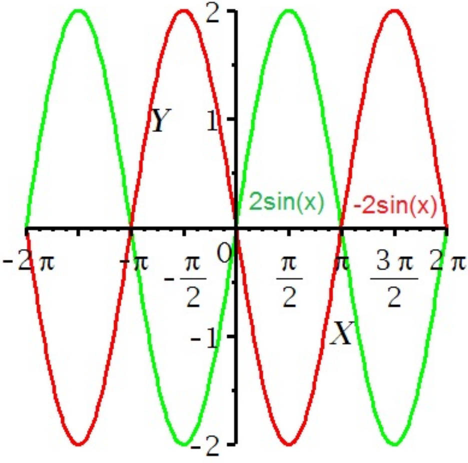
1. We look at $y = A\sin(x)$, $y = A\cos(x)$.
2. We look at $y = \sin(Bx)$, $y = \cos(Bx)$.
3. We look at $y = \sin(x + C)$, $y = \cos(x + C)$.
4. We combine these results.

Controlling the amplitude $|A|$.

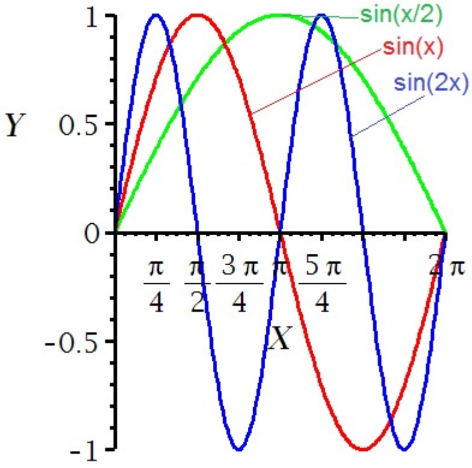
Recall that when we multiply a function by a constant, the y-values of the graph grow or shrink proportionally:



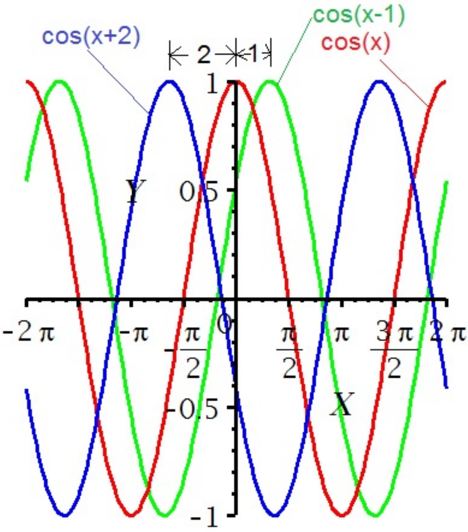
Negative A values also flip the graph across the horizontal axis.



Since $y = \sin(Bx)$ is multiplying the argument by B, this will change the period from $\tau = 2\pi$ to $\tau = 2\pi/B$.



Adding C to the argument shifts left or right.



Consider combining the results as follows...

Given $y = A\sin(Bx + C)$, we first factor out B inside the parentheses:

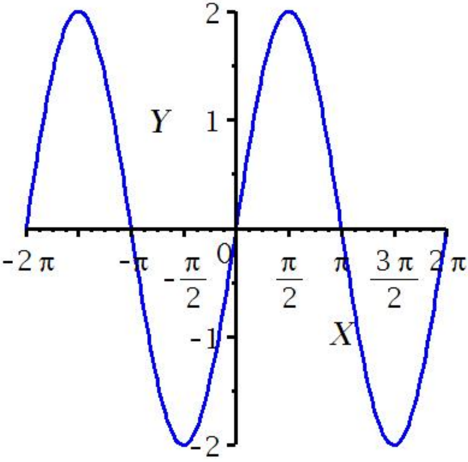
$$y = A\sin(Bx + C) = A\sin(B(x + C/B)).$$

Thus we note that the graph may be obtained in three steps:

1. Start with the graph of $y = \sin(x)$.
2. Change the amplitude; $y = A\sin(x)$.
3. Change the period to $\tau = 2\pi/B$; $y = A\sin(Bx)$.
4. Shift left/right depending on C/B; $y = A\sin(B(x + C/B))$.

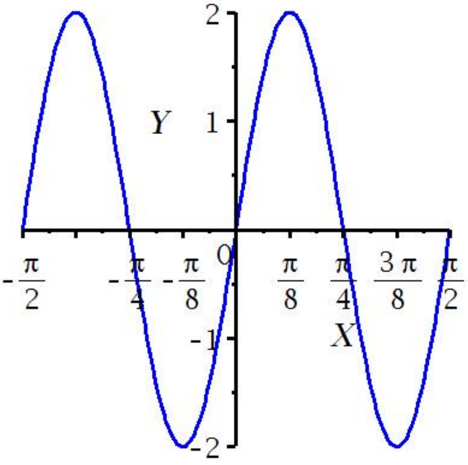
Example L14.1: Graph $y = 2\sin(4x - 8)$.

Solution: We write this as $y = 2\sin(4(x - 2))$. Let us see what happens step-by-step; first look at $y = 2\sin(x)$:



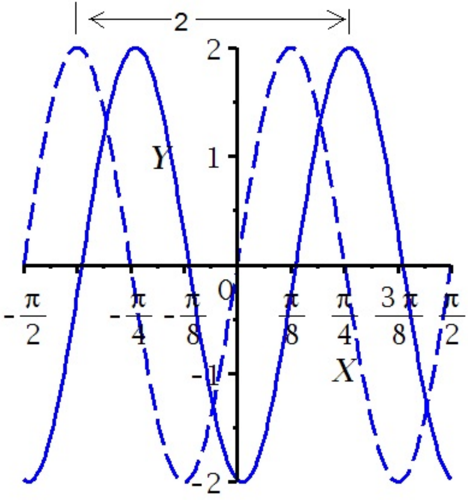
Example L14.1: Graph $y = 2\sin(4x - 8)$.

Solution: Next, look at $y = 2\sin(4x)$:



Example L14.1: Graph $y = 2\sin(4x - 8)$.

Solution: Finally, $y = 2\sin(4(x-2))$ is shifted right 2 units:

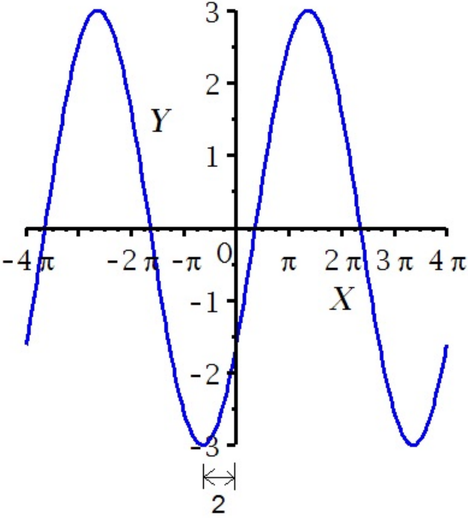


Example L14.2: Graph $y = -3\cos(x/2 + 1)$.

Solution: We write $y = -3\cos((x + 2)/2)$. Then we

1. Start with $\cos(x)$ and
2. flip across the horizontal axis,
3. rescale the y-axis by a factor of 3,
4. rescale the x-axis since the period is now $\tau = 2\pi/(1/2) = 4\pi$
5. and then shift left 2 units.

Here is the result.



Practice!

Problem L14.1: Plot $y = \frac{1}{4}\sin(2x + \pi/3)$.

Problem L14.2: Plot $y = 3\cos(x/2 - \pi/4)$.