Math 1060Q Lecture 12

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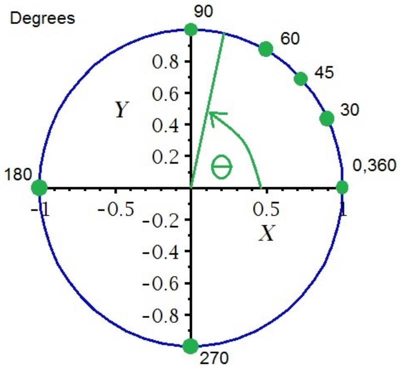
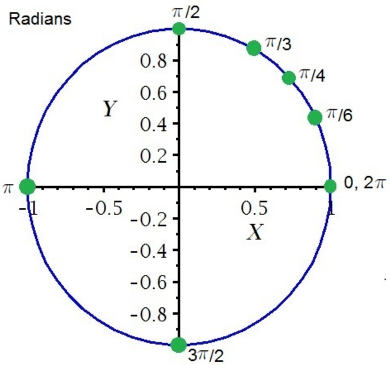
Today we get into trigonometry.

- Angle measures: degrees and radians
- Arc length and circular sectors
- Right triangles and trigonometric functions

The two main "units" for angle measures are degrees and radians.

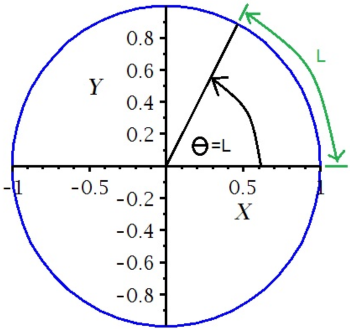
- We divide a circle into 360 degrees.
- We divide a circle into 2π radians.



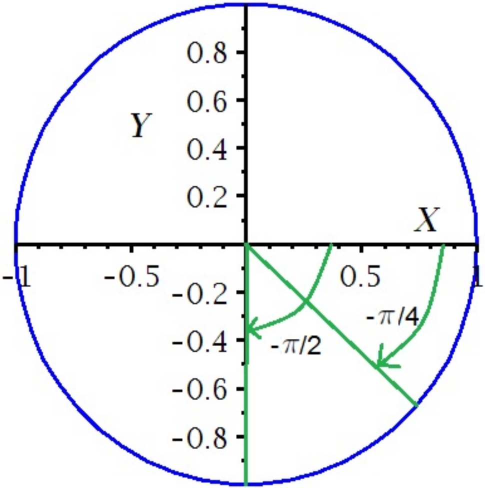


The radian measurement is the fraction of the perimeter of the unit circle.

- A circle of radius 1 has circumference 2π.
- The angle in radians is the lenth of the perimeter traced out along the unit circle, e.g. if we go $1/4$ of the way around we trace out an angle of $2\pi/4 = \pi/2$ radians.



Negative angles may be measured clock-wise.



Conversion of degrees to radians or radians to degrees.

- Note that $360^\circ = 2\pi$ rad, so we have

$$\frac{360^\circ}{2\pi \text{ rad}} = 1 = \frac{2\pi \text{ rad}}{360^\circ}.$$

- If preferred, you can reduce as follows:

$$\frac{180^\circ}{\pi \text{ rad}} = 1 = \frac{\pi \text{ rad}}{180^\circ}.$$

- To convert one way or the other, think of "cancelling units";

$$90^\circ = 90^\circ \frac{\pi \text{ rad}}{180^\circ} = \frac{\pi}{2} \text{ rad}.$$

$$\frac{\pi}{6} \text{ rad} = \frac{\pi}{6} \text{ rad} \frac{180^\circ}{\pi \text{ rad}} = 30^\circ.$$

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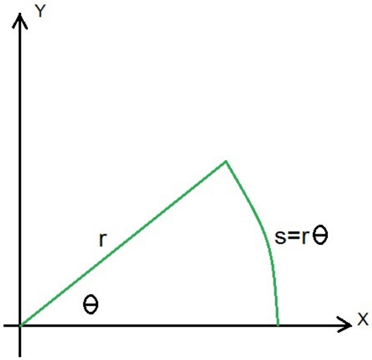
Arc length: a fraction of the circumference of a circle.

We already know that radian measure denotes arc length along a unit circle. If the radius of the circle is NOT $r = 1$, there is still a simple way to get the arc lenth, s: $s = r\theta$. But θ is in RADIANS here.



Area of a circular sector works in a similar way.

- Area of a circle: πr^2.
- Let F be some *fraction* of a circle swept out by some angle θ; the area of the corresponding circular sector is $F\pi r^2$.
- F can be found as follows:

$$\text{radians}: \quad F = \frac{\theta}{2\pi}, \quad \text{degrees}: \quad F = \frac{\theta}{360}.$$

- In summary, the area of a circular sector is

$$\text{radians}: \quad \frac{\theta}{2\pi}\pi r^2 = \frac{\theta}{2}r^2, \quad \text{degrees}: \quad \frac{\theta}{360}\pi r^2.$$

- It is probably easier to remember the derivation of these formulas than the formulas themselves.

Example

Example L12.1: What are the arc length and area of the circular sector swept out by an angle of $\theta = 135^\circ$ with radius $r = 10$?
Solution: We want radians for the arc length, so first convert:

$$135^\circ = 135^\circ \frac{\pi \text{ rad}}{180^\circ} = \frac{3\pi}{4} \text{ rad.}$$

(Make a picture to help if your algebra is not strong enough.)
Then apply the formulas:

$$s = r\theta = 10\frac{3\pi}{4} = \frac{15\pi}{2}.$$

$$\text{Area } = F\pi r^2 = \frac{3\pi/4}{2\pi}\pi 10^2 = 100\frac{3\pi}{8} = \frac{75\pi}{2}.$$

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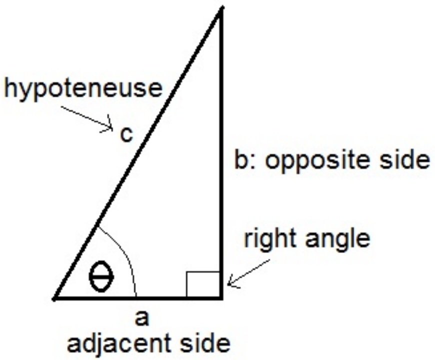
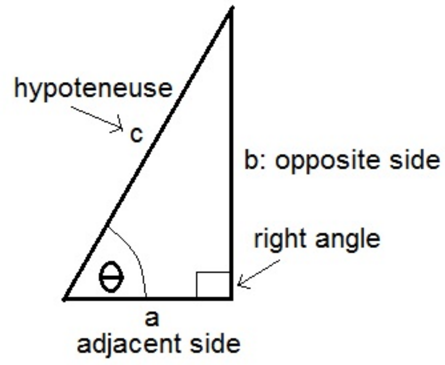
The primary trig. functions.



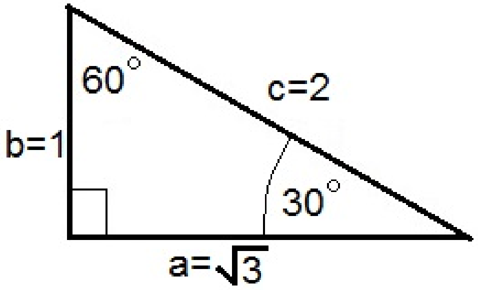
- $\sin(\theta) = \frac{b}{c}$ "opposite over hypoteneuse"
- $\cos(\theta) = \frac{a}{c}$ "adjacent over hypoteneuse"
- $\tan(\theta) = \frac{b}{a}$ "opposite over adjacent"

The trig. co-functions.



- $\csc(\theta) = \frac{c}{b} = \frac{1}{\sin(\theta)}$ "reciprocal of $\sin(\theta)$"
- $\sec(\theta) = \frac{c}{a} = \frac{1}{\cos(\theta)}$ "reciprocal of $\cos(\theta)$"
- $\cot(\theta) = \frac{a}{b} = \frac{1}{\tan(\theta)}$ "reciprocal of $\tan(\theta)$"

Special triangles: $30^\circ - 60^\circ - 90^\circ$

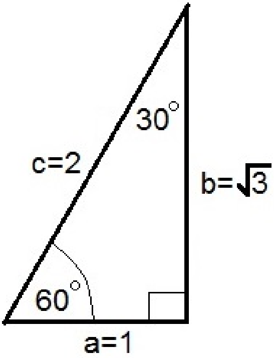


$$\sin(30^\circ) = \frac{1}{2} \qquad \csc(30^\circ) = \frac{2}{1} = 2$$

$$\cos(30^\circ) = \frac{\sqrt{3}}{2} \qquad \sec(30^\circ) = \frac{2}{\sqrt{3}}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \qquad \cot(30^\circ) = \sqrt{3}$$

Special triangles: $30^\circ - 60^\circ - 90^\circ$

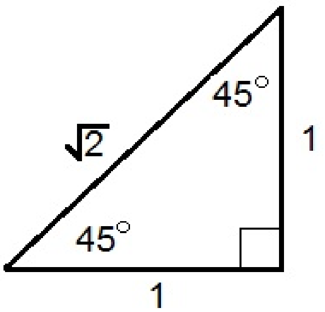


$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \csc(60^\circ) = \frac{2}{\sqrt{3}}$$

$$\cos(60^\circ) = \frac{1}{2} \qquad \sec(60^\circ) = 2$$

$$\tan(60^\circ) = \sqrt{3} \qquad \cot(60^\circ) = \frac{1}{\sqrt{3}}$$

Special triangles: $45^\circ - 45^\circ - 90^\circ$



$$\sin(45^\circ) = \frac{1}{\sqrt{2}} \qquad \csc(45^\circ) = \sqrt{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}} \qquad \sec(45^\circ) = \sqrt{2}$$

$$\tan(45^\circ) = 1 \qquad \cot(45^\circ) = 1$$

Examples

Example L12.2: If a 30°-60°-90° triangle has hypoteneuse 8 units long, find the lengths of the remaining sides.
Solution: Let x denote the length of the side opposite the 30° angle. Then

$$\sin(30^\circ) = \frac{x}{8} = \frac{1}{2} \Rightarrow x = \frac{8}{2} = 4.$$

Similarly, if y is the length of the remaining side,

$$\cos(30^\circ) = \frac{y}{8} = \frac{\sqrt{3}}{2} \Rightarrow y = \frac{8\sqrt{3}}{2} = 4\sqrt{3}.$$

Examples

Example L12.3: Someone looks up to the top of a building, which is 400 feet high, tilting their head 60° up to do so. How far away from the building are they?

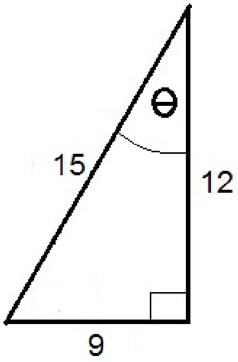
Solution: We have the relationship

$$\tan(60^\circ) = \frac{400}{x} = \sqrt{3} \Rightarrow x = \frac{400}{\sqrt{3}},$$

measured in feet.

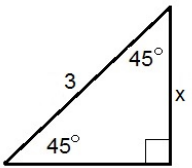
Practice!

Problem L12.1: Find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\sec(\theta)$, $\csc(\theta)$ and $\cot(\theta)$:



Practice!

Problem L12.2: Calculate x:



Problem L12.3: Calculate x: