Math 1060Q Lecture 1

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Welcome to PreCalculus!

The class website will be useful:

`www.math.uconn.edu/~connors/math1060f14/index.html`

Make sure you understand the syllabus.

- Most classes have quizzes at the beginning after a quick review.
- Homework is weighed heavily; this is where most of the work happens.
- Homework uses the online WebAssign system. You will be e-mailed a class key to register for it.

WebAssign website:

`www.webassign.net`

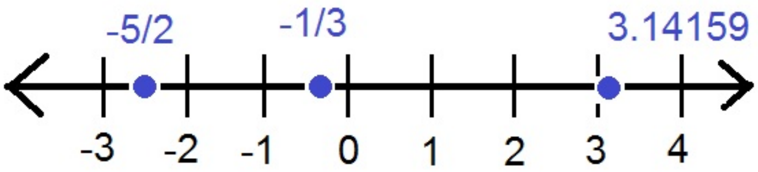
You have homework posted already.

Today's topic: real number system.

1. The real line.
2. Inequalities.
3. Intervals and sets.
4. Unions and intersections.
5. Absolute values.

The real line is just a way to visualize numbers.

The notation $\mathbb{R}$ means all real numbers. We may visualize particular numbers by labeling their position along a line:



- Tick marks are used to set a scale and points of reference.
- Points are labeled with a dot.
- The value "0" on the line is the *origin*.

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Inequalities are used to compare numbers.

Let a and b denote two real numbers. Then precisely one of the following must hold:

1. $a < b$: a is less than b and a appears to the left of b on the number line.
2. $a > b$: a is greater than b and a appears to the right of b on the number line.
3. $a = b$: a and b have the same value and are coincident on the number line.

- In case we know that $a < b$ OR $a = b$ (but are not sure which), we say $a \leq b$.
- Similarly, we may say $a \geq b$ if $a > b$ OR $a = b$.

We may wish to multiply both sides of an inequality by some value.

If we multiply/divide both sides of an inequality by a positive number, the inequality does not change:

$$\begin{aligned} -2 &< 3 \\ \Rightarrow 4(-2) &< 4(3) \\ \Rightarrow -8 &< 12. \end{aligned}$$

However, if we multiply/divide both sides of an inequality by a negative number, the inequality flips:

$$\begin{aligned} -2 &< 3 \\ \Rightarrow (-4)(-2) &> (-4)(3) \\ \Rightarrow 8 &> -12. \end{aligned}$$

You must remember when to flip the inequality to solve some problems.

Example L1.1: Find all values of x satisfying

$$4x - 10 < x + 20.$$

Solution:

$$\begin{aligned} 4x - 10 &< x + 20 \\ \Rightarrow 4x &< x + 30 \\ \Rightarrow 3x &< 30 \\ \text{(don't flip inequality)} \quad \Rightarrow x &< \frac{30}{3} = 10. \end{aligned}$$

You must remember when to flip the inequality to solve some problems.

Example L1.2: Find all values of x satisfying

$$x - 10 < 3x + 10.$$

Solution:

$$\begin{aligned} x - 10 &< 3x + 10 \\ \Rightarrow x &< 3x + 20 \\ \Rightarrow -2x &< 20 \\ \text{(flip inequality)} \quad \Rightarrow x &> \frac{20}{-2} = -10. \end{aligned}$$

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An open interval is a set of all numbers between two values, not including the bounding values.

The follwing are equivalent, just two notations:

$$(a, b) \quad \text{"interval notation"}$$

and

$$\{\ x\ |a < x < b\} \quad \text{"set notation"}.$$

This set does not include the values a or b; graphically we denote this using open circles and shade in between:

A closed interval is a set of all numbers between two values, including the bounding values.

The follwing are equivalent:

$$[a, b] \quad \text{"interval notation"}$$

and

$$\{\ x\ | a \leq x \leq b\} \quad \text{"set notation"}.$$

This set includes the values a and b; graphically we denote this using closed circles and shade in between:

A half-open interval means one of the two bounding values is included in the interval.

For example,

$$[a, b)$$

is the same as

$$\{\ x\ |a \leq x < b\}\,.$$

Visualization:

We may also have unbounded intervals.

This set is unbounded above:

$$(a, \infty) = \{\ x\ |a < x < \infty\}\,.$$

This set is unbounded below:

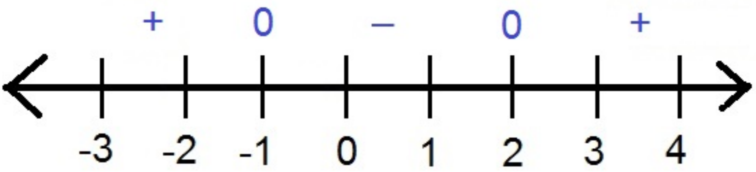
$$(-\infty, b] = \{\ x\ |-\infty < x \leq b\}\,.$$

Application: we use sign charts to solve some inequalities

When we have a product of terms and we compare to zero, a sign chart helps visualize where the product is positive or negative.

Example L1.3: Find all values of x such that $x^2 - x - 2 < 0$.
Solution:

1. First factor the expression and get $(x+1)(x-2) < 0$.
2. Visualize where the product is zero: at $x = -1$ and $x = 2$.
3. In between the zeroes, determine if the product is positive or negative and mark this information on the sign chart.



The inequality holds on the open interval $(-1, 2)$.

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Unions add sets together, intersections find where they overlap.

Consider the intervals $(a, b]$ and $[c, d)$. Their union is denoted by:

$$(a, b] \cup [c, d) = \{ x \mid a < x \leq b \text{ OR } c \leq x < d \} .$$

However, to be in the intersection of these two sets, x must be in both sets separately:

$$(a, b] \cap [c, d) = \{ x \mid a < x \leq b \text{ AND } c \leq x < d \} .$$

Example L1.4: Find all values of x such that $5x < 10$ and $-2x < 4$ both hold.
Solution: We require that $x < 2$ and that $x > -2$. In other words,

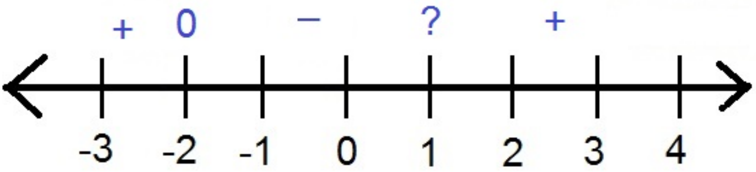
$$(-\infty, 2) \cap (-2, \infty) = (-2, 2).$$

Application: we use sign charts to solve some inequalities

A sign chart also helps when we compare a ratio of terms to zero.

Example L1.5: Find all values of x such that $\frac{x+2}{x-1} \geq 0$.
Solution:

1. Visualize where the numerator is zero: at $x = -2$.
2. Visualize where the denominator is zero: at $x = 1$ (mark using "?", since the ratio does not exist here).
3. In between the 0's and ?'s, determine if the ratio is positive or negative and mark this information on the sign chart.



The inequality holds on the set $(-\infty, -2] \cup (1, \infty)$.

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The absolute value signs just mean "make what is inside non-negative"

- We denote the absolute value of x by $|x|$.
- If x is negative, then $|x|$ means "make it positive"; e.g. $|-3| = 3$.
- If x is not negative, then $|x|$ means "just x itself"; e.g. $|2| = 2$ and $|0| = 0$.

The distance between two numbers, say a and b, is denoted by $d(a, b)$ and may be calculated as $d(a, b) = |a - b|$.
Properties of the absolute value:

- $|ab| = |a||b|$.
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$.
- $\{\, x \mid |x| < a\} = \{\, x \mid -a < x < a\}$.
- $\{\, x \mid |x| > a\} = \{\, x \mid x < -a \text{ OR } a < x\}$.

The last two properties are often used to solve problems.

Example L1.6: Find all x satisfying $|5x - 3| < 2$.
Solution: In this case, we must have

$$-2 < 5x - 3 < 2 \Rightarrow 1 < 5x < 5 \Rightarrow \frac{1}{5} < x < 1.$$

Example L1.7: Find all x satisfying $|5x - 3| > 2$.
Solution: In this case, we must have

$$5x - 3 < -2 \quad \text{OR} \quad 2 < 5x - 3.$$

Solve each of these inequalities to yield $x < 1/5$ or $x > 1$. Since either condition may hold, the answer is the union of these two intervals:

$$(-\infty, 1/5) \cup (1, \infty).$$

Practice!

Practice L1.1: Use set notation to express $(-1, 2]$.

Practice L1.2: Use interval notation to express

$$\{\ x \mid x < 0 \text{ OR } x > 5\}.$$

Practice L1.3: Find all x such that $x^2 - x - 6 \leq 0$.

Practice L1.4: Express $(-21, 5) \cap [3, 7]$ as a single interval.

Practice L1.5: Solve $|x - 1| \leq 2$.

Practice L1.6: Solve $|2x + 8| \geq 4$.