Math 1060Q: Exam 2 Review

Jeffrey Connors

University of Connecticut

November 3, 2014

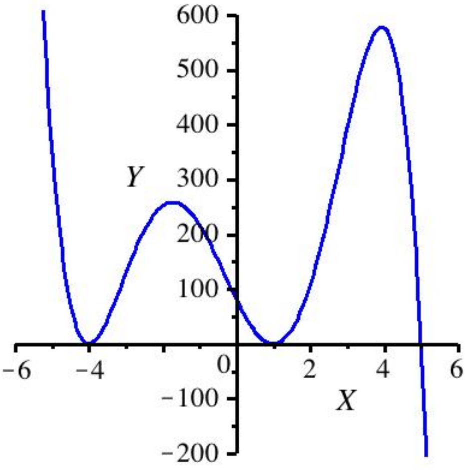
Problem 1

Find a polynomial $p(x)$ such that $p(x) \to -\infty$ as $x \to \infty$ with roots and corresponding multiplicities given by:

root	$x = 5$	$x = -4$	$x = 1$
multiplicity	1	2	2

Problem 1

A solution is $p(x) = -(x-5)(x+4)^2(x-1)^2$. Graph:



Problem 2

Write $p(x) = 3x^3 - 5x^2 - 11x - 3$ as a product of linear factors.
Hint: $x = -1$ is a root of $p(x)$.

Problem 2

Use long division to factor out $x + 1$:

$$
\begin{array}{r}
3x^2 - 8x - 3 \\
x + 1 \,\big)\, \overline{3x^3 - 5x^2 - 11x - 3} \\
-(3x^3 + 3x^2) \downarrow \quad \downarrow \\
\hline
-8x^2 - 11x - 3 \\
-(-8x^2 - 8x) \quad \downarrow \\
\hline
-3x - 3 \\
-(-3x - 3) \\
\hline
0
\end{array}
$$

Problem 2

So $p(x) = (x+1)(3x^2 - 8x - 3)$. We may use the quadratic formula to find any remaining roots (and hence linear factors):

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-3)}}{(2)(3)} = \frac{8 \pm \sqrt{64+36}}{6} = \frac{8 \pm 10}{6},$$

so $x = 3$ and $x = -1/3$ are both roots. Then we know

$$p(x) = a(x+1)(x-3)(x+1/3),$$

where we determine a by comparing with the form of $p(x)$, from which we see $a = 3$;

$$p(x) = 3(x+1)(x-3)(x+1/3).$$

Problem 3

Find and properly label/classify all asymptotes of:
(a) $p(x) = \frac{x^3-6x^2-x+1}{3x^4+1}$ and
(b) $p(x) = \frac{x^3+x^2-3x-3}{x^2-4}$.

Problem 3

(a) $p(x) = \frac{x^3-6x^2-x+1}{3x^4+1}$

Here the denominator is of higher-order than the numerator, hence $y = 0$ is a horizontal asymptote. There are no other asymptotes.

(b) $p(x) = \frac{x^3+x^2-3x-3}{x^2-4}$.

Vertical asymptotes will occur where $x^2 - 4 = 0$. Solving for x, we see that $x^2 = 4 \Rightarrow x = \pm 2$ are both vertical asymptotes. Since the numerator is one order higher than the denominator, there will be a slant asymptote. We use polynomial division to find it: $y = x + 1$ (see next slide).

Problem 3

$$
\begin{array}{r}
x+1 \\
x^2-4 \,\big)\, \overline{x^3+x^2-3x-3} \\
\underline{-(x^3-4x)} \\
x^2+x-3 \\
\underline{-(x^2-4)} \\
x+1
\end{array}
$$

$x+1$ ← this is the asymptote

remainder $x+1$ is lower-order than x^2-4

Problem 4

Fill in the table: (angles are in radians)

θ	$7\pi/6$	$\pi/4$
$\cos(\theta)$		
$\sin(\theta)$		
$\tan(\theta)$		
$\sec(\theta)$		
$\csc(\theta)$		

Problem 4

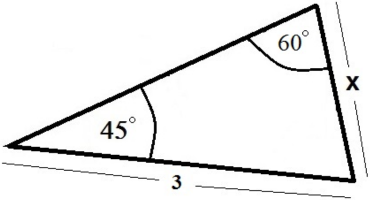
Fill in the table: (angles are in radians)

θ	$7\pi/6$	$\pi/4$
$\cos(\theta)$	$-\sqrt{3}/2$	$1/\sqrt{2}$
$\sin(\theta)$	$-1/2$	$1/\sqrt{2}$
$\tan(\theta)$	$1/\sqrt{3}$	1
$\sec(\theta)$	$-2/\sqrt{3}$	$\sqrt{2}$
$\csc(\theta)$	-2	$\sqrt{2}$

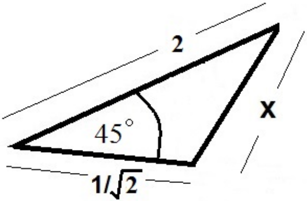
Problem 5

Calculate x as shown in each diagram:

(a)

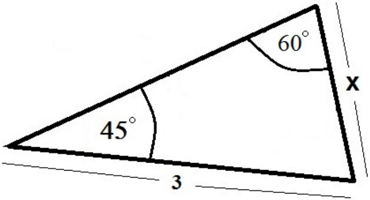


(b)



Problem 5

(a)



Use the Law of Sines...

$$\frac{\sin(60^\circ)}{3} = \frac{\sin(45^\circ)}{x} \Rightarrow \frac{\sqrt{3}}{6} = \frac{1}{x\sqrt{2}}.$$

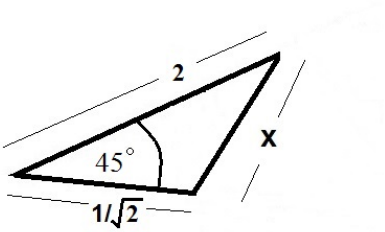
Solve for x:

$$x\sqrt{2} = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3},$$

so $x = 2\sqrt{3}/\sqrt{2} = \sqrt{6}$.

Problem 5

(b)



Law of Cosines...

$$x^2 = 2^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 2\frac{1}{\sqrt{2}}2\cos(45^\circ)$$

$$x^2 = 4 + \frac{1}{2} - \frac{4}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{5}{2}$$

$$\Rightarrow x = \sqrt{\frac{5}{2}}.$$