Math 1060Q: Exam 1 Review

Jeffrey Connors

University of Connecticut

September 24, 2014

Problem 1

Find all values of x that satisfy the inequalities below. Express your answers using interval notation.

(a) $|x - 5| > 3$.
Solution: we solve two inequalities; the first is

$$x - 5 > 3 \Rightarrow x > 8.$$

Then we solve

$$x - 5 < -3 \Rightarrow x < 2.$$

We take both answers together: $(-\infty, 2) \cup (8, \infty)$.

Problem 1

Find all values of x that satisfy the inequalities below. Express your answers using interval notation.

(b) $|-2x+4| \leq 2$.
Solution: we solve two inequalities "simultaneously":

$$-2 \leq -2x+4 \leq 2 \Rightarrow -6 \leq -2x \leq -2$$

and remember to flip the inequality on the final step...

$$3 \geq x \geq 1.$$

The answer is $[1,3]$.

Problem 2

Find the center and radius of the circle defined by $x^2 + x + y^2 - 4y + 9/4 = 0$.
Solution: complete the square...

$$x^2 + x + \left(\frac{1}{2}\right)^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{-4}{2}\right)^2 - \frac{9}{4}$$

$$\left(x + \frac{1}{2}\right)^2 + (y-2)^2 = \frac{1}{4} + 4 - \frac{9}{4}$$

$$\left(x + \frac{1}{2}\right)^2 + (y-2)^2 = \frac{1}{4} + \frac{16}{4} - \frac{9}{4} = \frac{8}{4} = 2.$$

The center is $(-1/2, 2)$ and the radius is $\sqrt{2}$.

Problem 3

Find the equation for the line passing through $(2, -3)$ that is:

(a) parallel to $5y + 10x = 1$.
Solution: the slope of the line we want is not clear; we must put the given line into a standard form:

$$5y = -10x + 1 \Rightarrow y = -2x + \frac{1}{5}.$$

Now we see the slope must be $m = -2$. We are given a point that the desired line goes through, so apply point-slope form:

$$y - (-3) = -2(x - 2) \Rightarrow y + 3 = -2(x - 2).$$

Problem 3

Find the equation for the line passing through $(2, -3)$ that is:

(b) perpendicular to $y = -3x + 7$.
Solution: Since the given line is in slope-intercept form, we see that a perpendicular line would have slope $m = -1/(-3) = 1/3$. Again, since we are given a point, we apply point-slope form:

$$y + 3 = \frac{1}{3}(x - 2).$$

Problem 4

Given the parabola $f(x) = 3x^2 + 18x + 11$, find:

(a) the standard form of the parabola
Solution: complete the square...

$$
\begin{aligned}
f(x) = 3\left(x^2 + 6x\right) + 11 &= 3\left(x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right) + 11 \\
&= 3\left((x+3)^2 - 9\right) + 11 \\
&= 3\left(x+3\right)^2 - 27 + 11 \\
&= 3\left(x+3\right)^2 - 16.
\end{aligned}
$$

Problem 4

Given the parabola $f(x) = 3x^2 + 18x + 11$, find:

(b) the domain and range for $f(x)$.
Solution: Think of the graph, based upon the standard form. It is a parabola, so the domain is always $(-\infty, \infty)$. The range depends on whether the parabola opens up or down; in this case it opens up. Thus, note the y-coordinate of the vertex (bottom of the parabola) is -16, hence the range is $[-16, \infty)$.

Problem 5

Given $f(x) = \frac{1}{x-5}$ and $g(x) = x^2 + 1$, find the domain of:

(a) $(f(g(x))$
Solution: Note this composition is

$$f \circ g = \frac{1}{x^2 - 4},$$

so we must not have $x^2 - 4 = 0$, i.e. $x \neq \pm 2$. The domain is $\{x \mid x \neq \pm 2\}$.

(b) $(g(f(x))$.
Solution: This composition is

$$g \circ f = \left(\frac{1}{x-5}\right)^2 + 1,$$

so we cannot have $x = 5$, otherwise there is no problem. Hence, the domain is $\{x \mid x \neq 5\}$.