Topics which will NOT be in the final: variation of parameters, predator-prey problem (section 8.2), Bessel equation (section 10.5), oscillation theorem (section 5.7), solution of some nonlinear 2nd order equations (section 5.8), wave equation. However the formula sheet will still retain these formula.

Likely there will be 5 questions in the exams (Of course there may be multiple parts for each question). Homework and quizzes are good sources for revision.

(1) Phase plane analysis (sections 8.3-8.4).
(2) Power series solution for linear homogeneous equations. Include ordinary points (section 10.3) and regular singular points (section 10.4; Frobenius method).

(3) eigenfunctions and heat equations with general initial conditions (sections 9.1 and 9.3). 
\(\phi'' + \lambda \phi = 0\) can be subjected to 4 different types of boundary conditions: one of the conditions \(\phi = 0\) or \(\phi_x = 0\) at \(x = 0\) combines with one of the conditions \(\phi = 0\) or \(\phi_x = 0\) at \(x = L\). (see homework Exercises 10, 11 on p.196).

The boundary conditions of heat equation will determine the boundary conditions of eigenfunction \(\phi\) being used. Able to tackle the case with general initial conditions.

(4) Definition of linearly independence of two functions \(\{f, g\}\). Definition of Wronskian of scalar and vector functions.

\[ W(f, g) = \det \begin{pmatrix} f & g \\ f' & g' \end{pmatrix}; \quad W(f, g) = \det (f, g). \]

Relation between Wronskian and linearly independence.

Second order linear homogeneous equations; Euler’s equation; reduction of order.

(5) General solution to system of equations (Chapter 7) \(\frac{dx}{dt} = Ax\) with \(A\) being a constant \(2 \times 2\) matrix. Pure imaginary eigenvalues of \(A\) gives rise to periodic solution.

Hamiltonian system (section 8.1): able to check if a system of equations is in fact a Hamiltonian system. Able to find the corresponding Hamiltonian.