Math 3160, Answer to Quiz 6 (2/29/12)

(1) Last week quiz: A biased coin has a probability 1/3 of getting a head when flipped. This coin is flipped until a total of 2 heads or 2 tails occur. (It is *not* necessary for the 2 heads or 2 tails to occur in consecutive flips.) Let X be the number of flips that is needed.

We already know that $X = 2$ or 3 and $E(X) = 22/9$. You need to find/recall $P\{X = 2\}$ and $P\{X = 3\}$.

(a) Compute $E(X^2)$.

(b) Compute $Var(X)$.

Answer: (a) $E(X^2) = 56/9$ (b) $\frac{56}{9} - (\frac{22}{9})^2$

Reasons: (a)

$$P\{X = 2\} = P(2 \text{ heads in the first 2 flips}) + P(2 \text{ tails in the first 2 flips})$$

$$= (1/3)^2 + (2/3)^2 = 5/9 \ .$$

Hence $P\{X = 3\} = 1 - P\{X = 2\} = 4/9$. So

$$E(X^2) = \sum x^2 P\{X = x\} = 2^2 \frac{5}{9} + 3^2 \frac{4}{9} = \frac{56}{9} \ .$$

(b) It follows that $Var(X) = E(X^2) - (E(X))^2 = \frac{56}{9} - (\frac{22}{9})^2$.

(2) Assume the probability of winning a game is 0.01 and you play a total of 100 (independent) games.

(a) What is the exact probability that you win at least 2 games?

(b) Using Poisson random variable to approximate the above probability, what is the approximate probability of winning at least 2 games?

Answer:

Reasons: (a) Let X be the number of games won. It is a binomial random variable with parameter $(100, 0.01)$. Hence the (exact) probability of winning at least 2 games is:

$$1 - P\{X = 0\} - P\{X = 1\} = 1 - (0.99)^{100} - \binom{100}{1} (0.01)^1 (0.99)^{99} \ .$$

(b) One can approximate a binomial random variable with parameter (n, p) by a Poisson random variable with parameter $\lambda = np$ when n is large and p is small.

Take $n = 100$ and $p = 0.01$. Thus $\lambda = 1$. So the approximate probability of winning at least 2 games is:

$$\begin{aligned} 1 - P\{X = 0\} - P\{X = 1\} &\approx 1 - e^{-1} - e^{-1}\frac{1}{1!} \\ &= 1 - 2e^{-1}\ . \end{aligned}$$