Calibration of a Regime-Switching Interest Rate Model

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Context for the Model

- Long-Rate Anchor: 20 Yr, Not (yet) Whole Curve
- Stress-testing
  - Not Forecasting
  - Not Pricing
- What's Important:
  - Severe but Plausible Extreme Scenarios
    - Plausible: in historical context
    - Severe: represent real stresses
    - Extreme: on both (all) tails
- Much Less Important:
  - Accuracy Around the Likely Scenarios
- Completely Irrelevant:
  - Risk Neutrality
  - Arbitrage Free

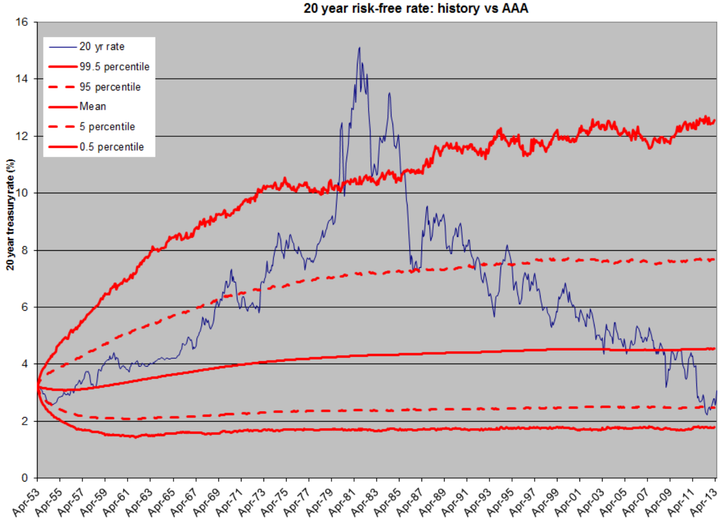
- Typical Generators (e.g. AAA).....
  - Gaussian-based volatility driver
  - A single mean reversion point (MRP)
- .....Fail To Produce Historically Plausible Ranges of Results
  - Unhistorical shape to the realized volatility
  - Tightly bunched paths versus historical ranges
  - MRP assumption largely drives the extreme paths
- To Fix the Problems
  - Use fat-tailed volatility driver
  - Randomize MRP to spread range of extreme paths
- But This Introduces More Parameters
  - Calibration becomes a real challenge

History of 20 Year US Treasury Rate

Plausible By Definition

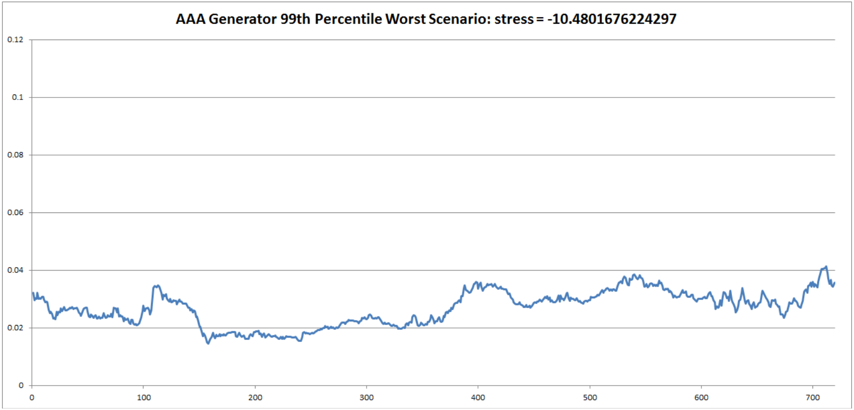
20 Yr Treasuries: History vs AAA Generator Monthly %-iles

Neither Early 80's Nor Japan Are Remotely Plausible In AAA

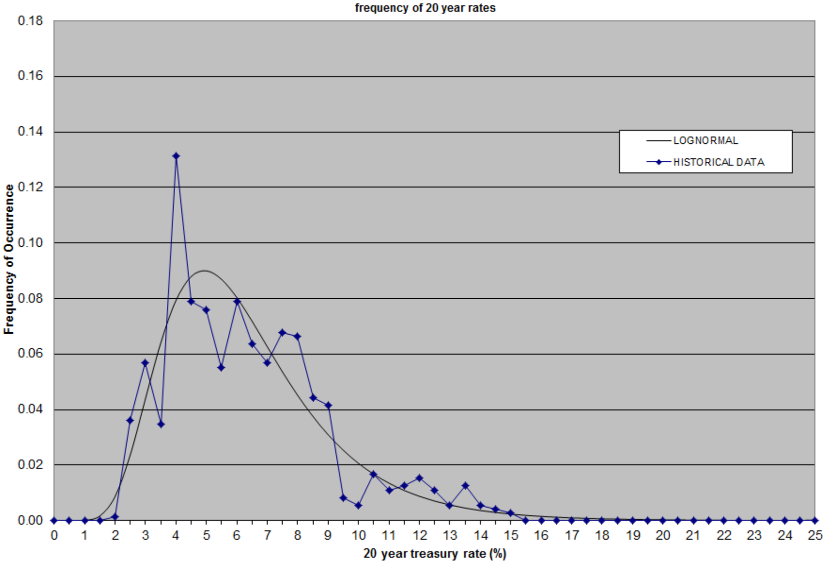


No One Path Follows the Monthly Extremes

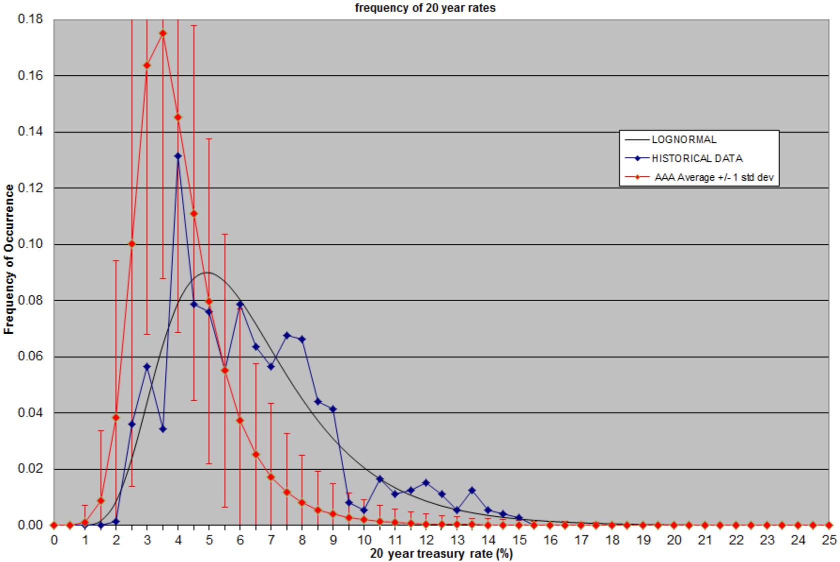
AAA Extreme Paths Are Not Japan-Like Near-Term - But They Persist



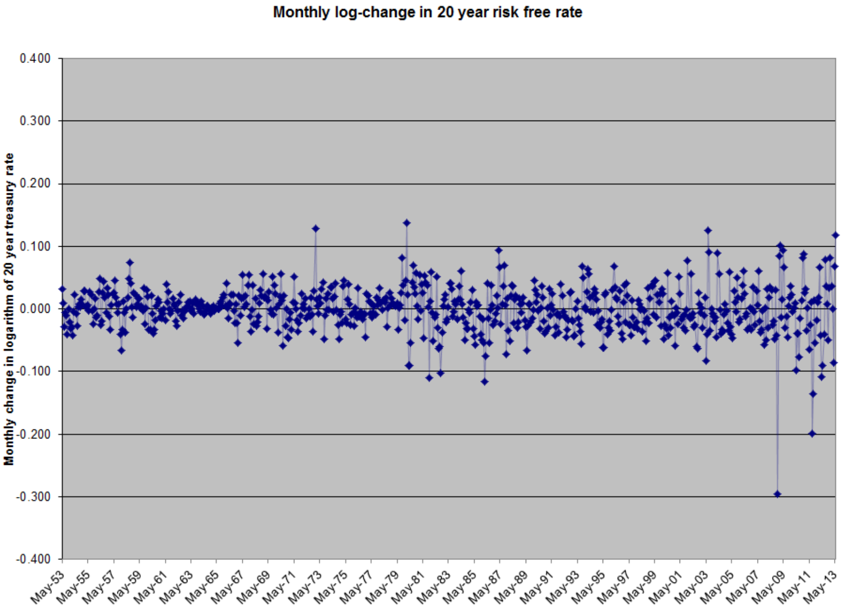
Historical Frequency of 20 Year Rates



Historical Frequency of 20 Year Rates vs AAA Generator

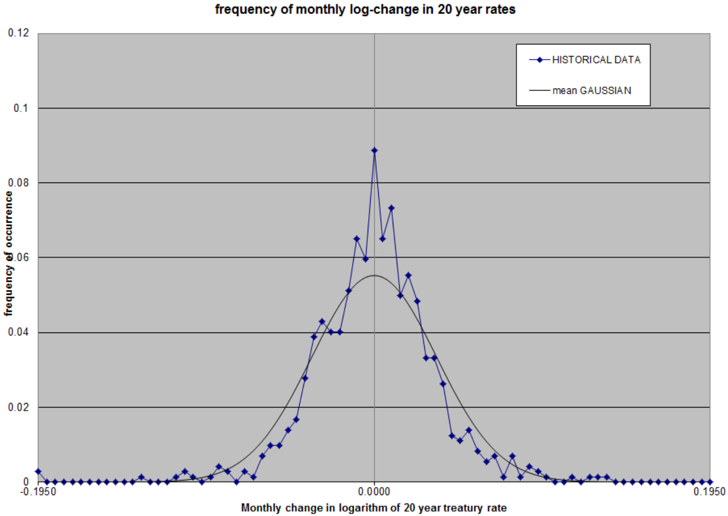


Historical Realized Volatility of 20 Year Rates



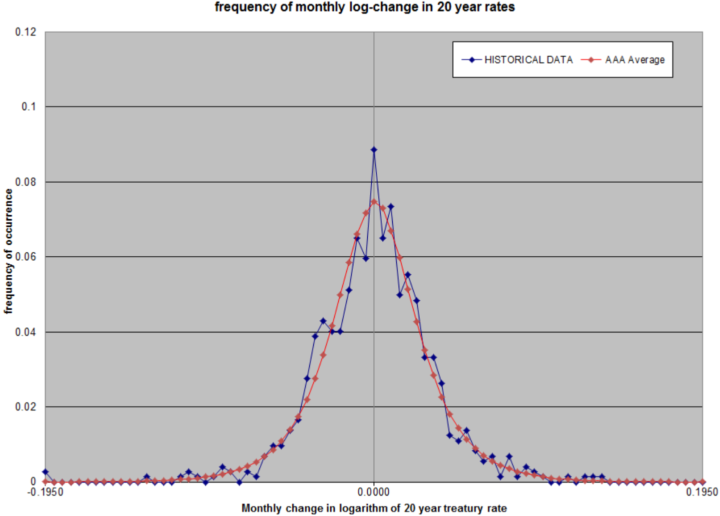
Historical Distr. of Realized Volatility of 20 Year Rates

High Kurtosis

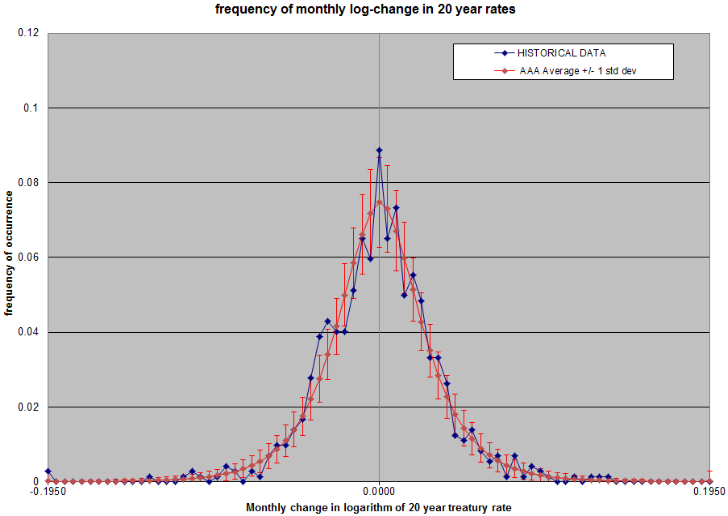


Historical Distr. of Realized Volatility vs AAA Generator

Stochastic Volatility Helps, May Not Fully Pick Up The Tails



Missing Tails Are Significant



Comparative Statistics: History vs AAA

Rate Levels and Spread as well as the Shape of the Realized Volatility Differ Significantly from History

	60 Year History	AAA Mean	AAA StdDev
Rate = 20 Year Treasury			
Rate Mean	.0635	**.0410**	.0081
Rate StdDev	.0266	**.0117**	.0058
Rate Kurtosis (normal=3)	3.53	3.02	1.29
Rate 6th-osis (normal=15)	21.5	17.7	26.1
(6th Ctrl Mom/StdDev^6)			
Realized Volatility = $\Delta \ln$Rate			
Volatility StdDev	.0360	.0338	.0039
Volatility Kurtosis (normal=3)	10.9	**5.3**	1.6
Volatility 6th-osis (normal=15)	479	**76**	124

- Traditional Models (including AAA)

$\Delta \ln Rate_t =$
$F * (\ln MRP - \ln Rate_{t-1}) + SlopeAdjustment + (1 - F) * Gaussian\Delta$

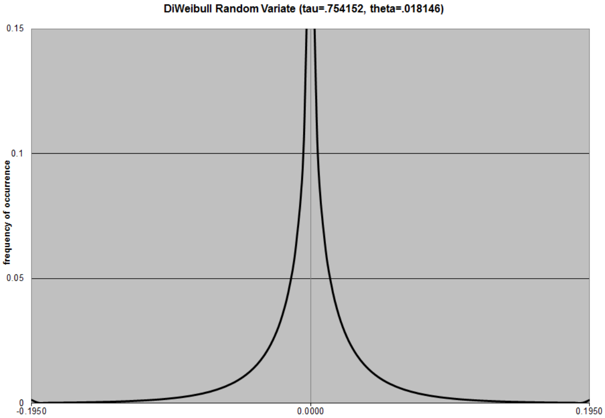
- Proposed New Model:

Regime-Switching with Random Regimes

$\Delta \ln Rate_i =$
$F * (\ln MRP_t - \ln Rate_{t-1}) - DriftCompensation + (1 - F) * DiWeibull\Delta$
where
$MRP_t = MRP_{t-1}$
unless
$t - t_{regime} >$ a random $Gamma(\alpha, \beta)$ variate.
In that case, the regime switches to a new, random MRP:
$MRP_t =$ a random $LogNormal$ variate, fixed until next regime-switch.
And the regime-switching clock restarts at $t_{regime} = t$.
(a $SlopeAdjustment$ can be included if desirable)

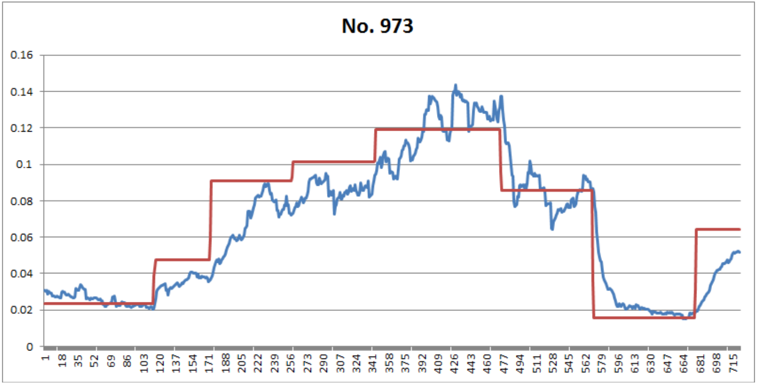
What Is A DiWeibull?

DiWeibull Is Like Laplace:
Laplace is symmetric Exponential, DiWeibull is symmetric Weibull



Very Heavy Tail

A Sample Path From the New Model (inti-MRP 4-53)



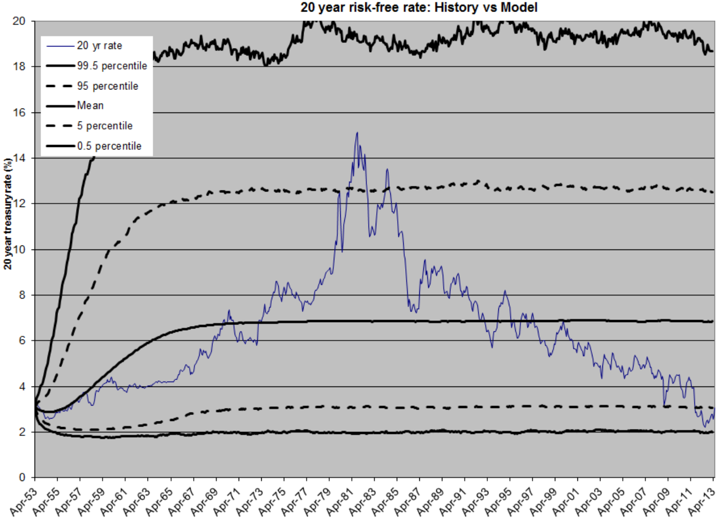
New Model Requires 8 Parameters

- 2 Parameters For The Regime Clock Random $Gamma(\alpha, \beta)$ Variate.
  - $\alpha = 7.1$ and $\beta = 1.14$ (in annualized units) follows from MLE applied to historical random MRP estimates derived by Least Square Error analysis versus historical rates
  - Average length of an interest rate regime is $\alpha\beta = 8$ Years plus 1 Month
- 1 Initial Value For The MRP
  - Least Square Error analysis versus historical rates gives
    - For 4-1953 start: init-MRP=2.36%
    - For 6-2013 start: init-MRP=2.04%
- This Leaves 5 Parameters To Be Determined
  - 2 Parameters For The Lognormal Random MRP
  - 2 Parameters For The *DiWeibull*Δ Volatility Driver
  - 1 Mean Reversion Strength Factor (F in the formula)
- Choose The 5 Parameters To Best Align Comparative Statistics vs History

Rate Levels and Spread as well as the Shape of the Realized Volatility Now Align With History

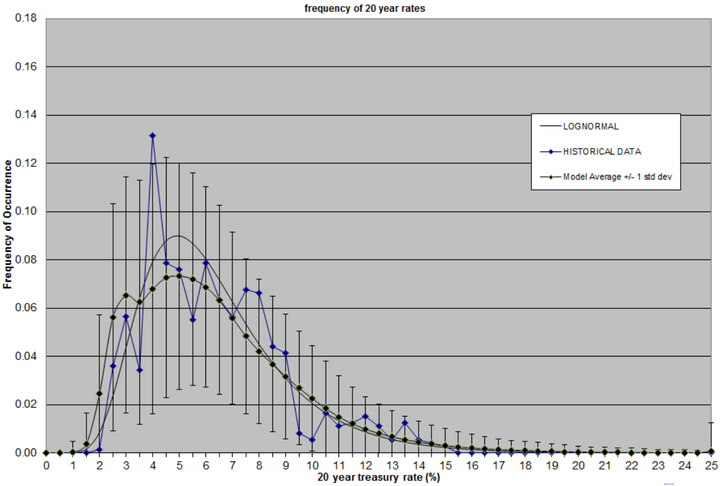
	60 Year History	**Model Mean**	**Model StdDev**
Rate = 20 Year Treasury			
Rate Mean	.0635	**.0631**	.0126
Rate StdDev	.0266	**.0268**	.0105
Rate Kurtosis (normal=3)	3.53	2.96	1.24
Rate 6th-osis (normal=15)	21.5	15.8	18.9
(6th Ctrl Mom/StdDev^6)			
Realized Volatility = Δ lnRate			
Volatility StdDev	.0360	.0363	.0027
Volatility Kurtosis (normal=3)	10.9	**10.9**	4.8
Volatility 6th-osis (normal=15)	479	**365**	636

New Model (init-MRP 4-53) vs History: Monthly %-iles

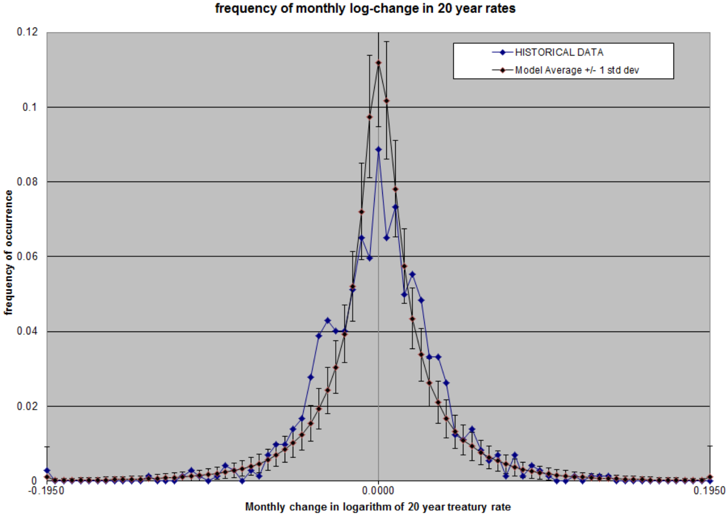


Only 55/723 Months Breach 5%-95%: History Fits Into This Easily

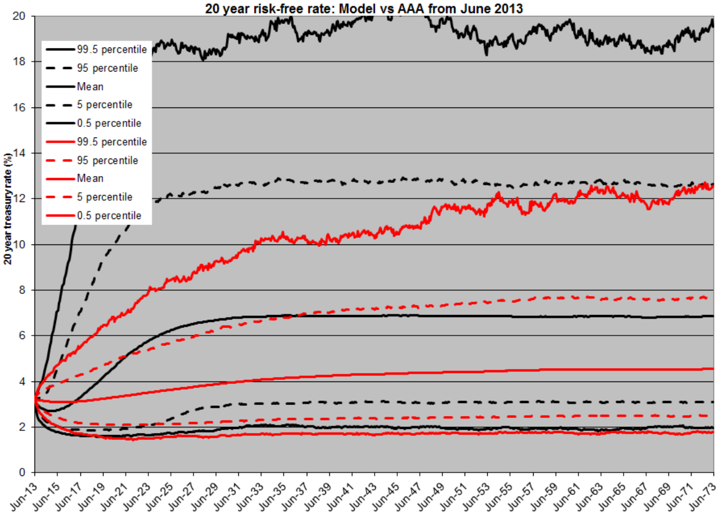
Fits Like A Glove



Too Far In The Other Direction? At Least The Tail Is Good

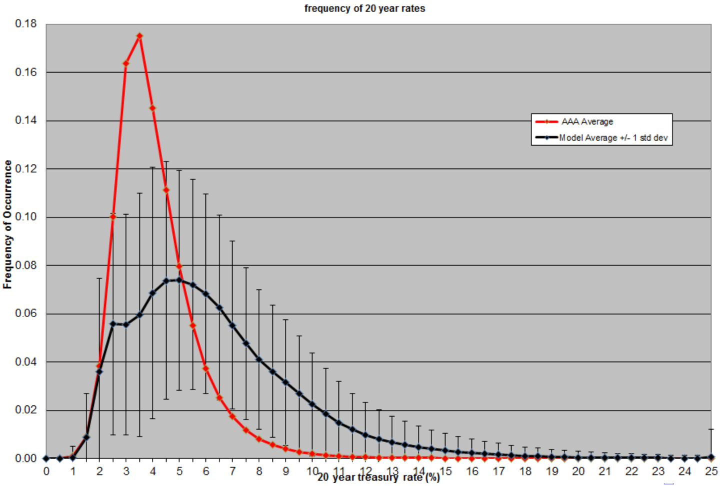


AAA Vs New Model (init-MRP 6-13): Monthly %-iles



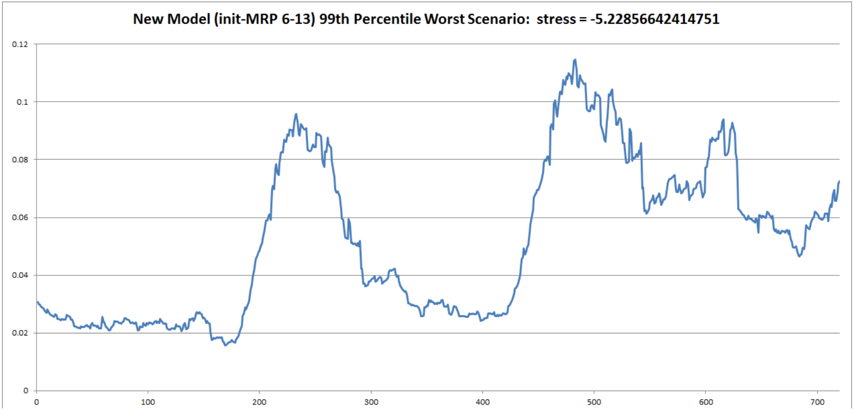
AAA Vs New Model (init-MRP 6-13): Rate Frequency

Same Prob. $\leq$ 2.25%, Wild Difference Thereafter



An Extreme Path In The New Model (init-MRP 6-13)

For First 15 Years Slightly More Stress Than The 99%-ile AAA Scenario (And After 15 It Has Different Stresses That AAA Would Never Generate)

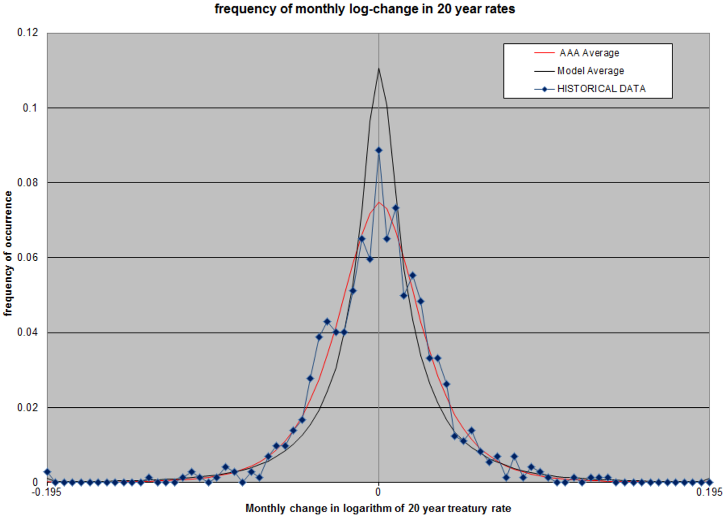


Shape Of Model Realized Volatility Is Not Only Fatter-Tailed On Average But Also Much More Varied

	Model Mean	**Model StdDev**	**AAA Mean**	**AAA StdDev**
Rate = 20 Year Treasury				
Rate Mean	.0628	.0126	.0410	.0081
Rate StdDev	.0271	.0104	.0117	.0058
Rate Kurtosis (normal=3)	2.94	1.19	3.02	1.29
Rate 6th-osis (normal=15)	15.3	17.7	17.7	26.1
(6th Ctrl Mom/StdDev^6)				
Realized Volatility = $\Delta \ln$Rate				
Volatility StdDev	.0364	.0027	.0338	.0039
Volatility Kurtosis (normal=3)	10.8	**5.0**	5.3	**1.6**
Volatility 6th-osis (normal=15)	368	**706**	76	**124**

Realized Vol: New Model (init-MRP 6-13) vs AAA

Both Miss Parts of Historical Volatility Shape Despite Other Evidence



Instead of Kurtotis and 6th-osis:

- Minimize L2 Distance of CDF to History

$$\sqrt{\int (F(r) - H(r))^2 \, dr}$$

- Minimize L1 Distance of CDF to History

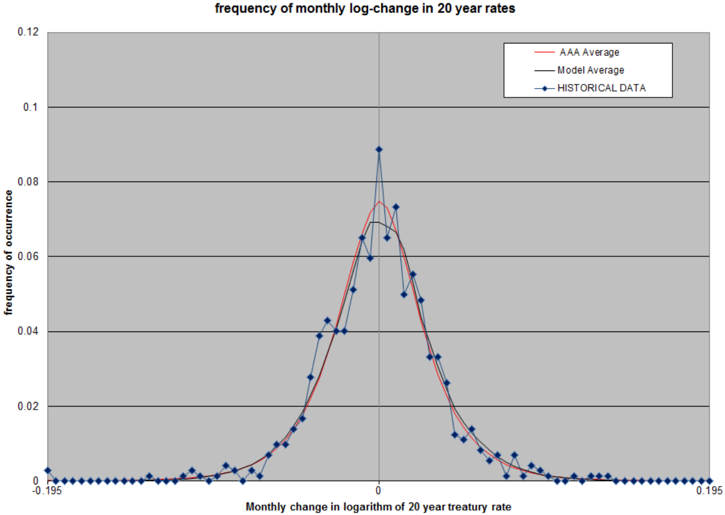
$$\int |F(r) - H(r)| \, dr$$

- Use CDF Rather Than PDF To Emphasize Tails
- Use Both Rates and Realized Volatility

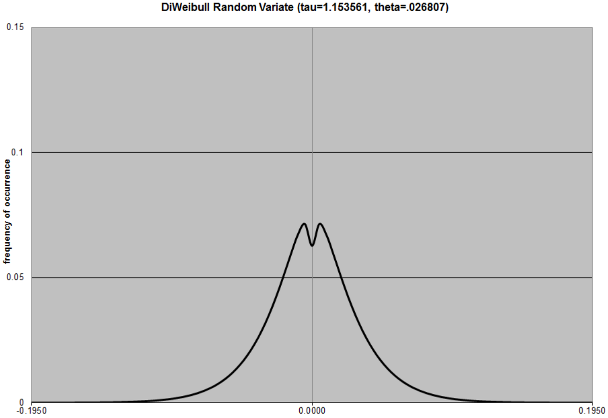
Calibration On L2 and L1 Distance, Means, Vol Std Dev

	Model Mean	Model StdDev	AAA Mean	AAA StdDev
Rate = 20 Year Treasury				
Rate Mean	.0631	.0078	.0410	.0081
Rate StdDev	.0190	.0048	.0117	.0058
L2 Distance to History	.0372	.0135	.0858	.0271
L1 Distance to History	.0102	.0035	.0230	.0070
Realized Volatility = $\Delta \ln$Rate				
Volatility StdDev	.0335	.0018	.0338	.0039
L2 Distance to History	.0067	.0012	.0074	.0030
L1 Distance to History	.0027	.0004	.0031	.0013

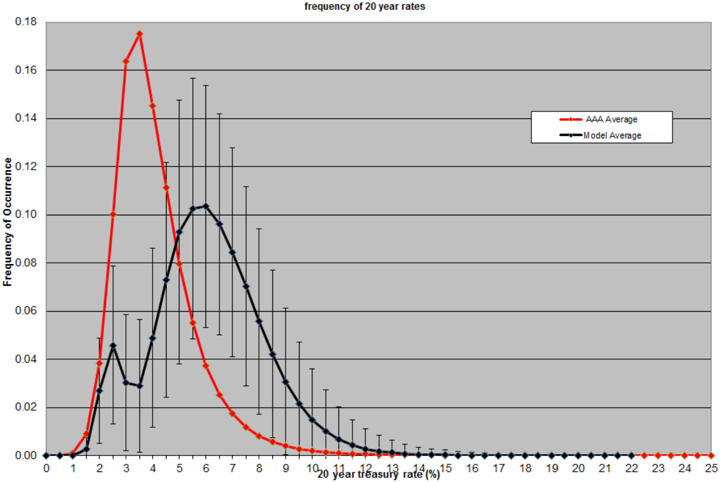
Realized Vol. Comparison For This Alternative Calibration



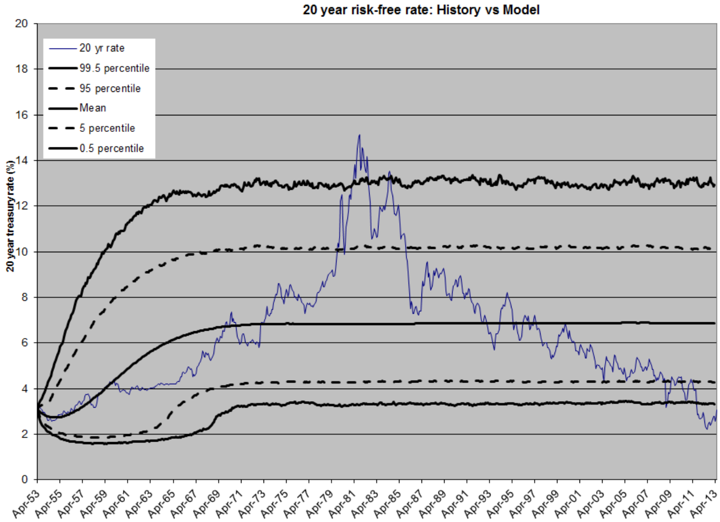
With This Calibration The Volatility Driver Has Milder Tail
BiModal Not A Problem: Mean-Reversion Smooths It Out In Realized Vol.



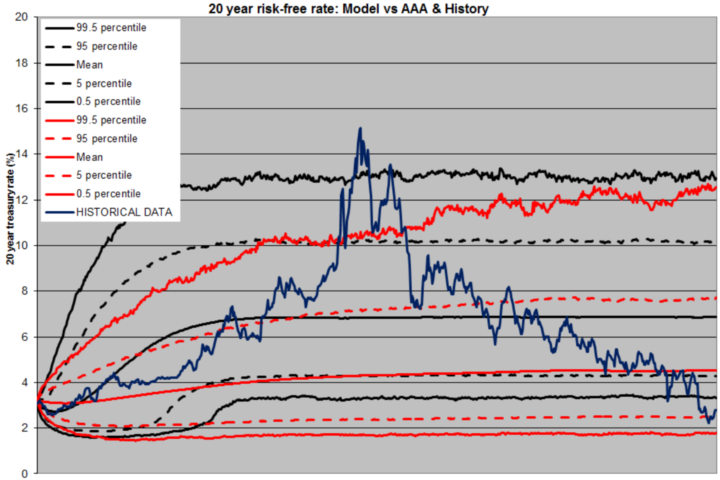
Rate Distr. Comparison For This Alternative Calibration



Monthly %-iles vs History For This Alternative Calibration



And Compared To AAA Generator



Extreme Path In This Alternative Calibration

Still Japan-like For A Good 15 Years