

Interest Rate Modeling With Random Regimes

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INTEREST RATES

STRESS TESTING

MODELS

20 Year U.S. Treasury Rates



ANECDOTAL HISTORY

1974: Guaranteed Income Contracts in the Group Pension Market



ANECDOTAL HISTORY

Early 1978: 12% Will Bring Blood In the Streets



ANECDOTAL HISTORY

February 1980: It Didn't; October 1981: Even 15.13% Didn't



FAILURE TO "SEE" A PLAUSIBILITY WAS CATASTROPHIC FOR INSURERS

- One Of The Largest Was Bankrupted
 - An Acquisition Disguised The Fact
- At Least One Of The Largest Visited The Fed
 - ... For A "Maybe, What If?" Discussion
 - Turned Out To Be Unnecessary

Saving Grace: Double-Digit Rates Only Lasted For 6 Years



SET TODAY'S STAGE

- Dynamic Valuation Interest Rates
- Asset Adequacy Analysis
- Appointed Valuation Actuary
- Principles-Based Discussion Began

ANECDOTAL HISTORY

January 1993: The Fed Won't Tolerate Long Rates Below 4%



ANECDOTAL HISTORY

January 1999: Maybe They Will, But That's What RBC Is For



ANECDOTAL HISTORY

January 2003: No, That's What Asset Adequacy Reserves Are For



FACTUAL HISTORY ~ FUTURE QUESTION

December 2008: How Long Under 4%?; June 2011: Measured From When?



- STRESS-TESTING CAN APPLY TO
 - Reserves
 - Asset Adequacy Testing
 - Risk Management for Product/Line of Business
 - Surplus
 - Risk Based Capital
 - Embedded Value
 - Economic Capital
 - Basel II (III, etc.)
 - Own Solvency and Risk
 - Enterprise Risk Management

STRESS-TESTING IS NOT

- A BY-PRODUCT OF FORECASTING
 - Forecasting Looks For Most Likely Outcomes
 - Maybe Within A Confidence Band
 - Forecasting Supports Current Decision-Making
 - Forecasting Will Be Judged By Actual Accuracy

STRESS-TESTING IS NOT

- A BY-PRODUCT OF PRICING
 - Pricing Looks For Expected Values
 - Usually With Reasonable Variance Bounds
 - Pricing Supports Product Portfolio Development
 - Pricing Will Be Judged By Average Accuracy

STRESS-TESTING IS

- A SEPARATE, DISTINCT DISCIPLINE
 - One That Looks For Extreme Values
 - Beyond Reasonable Variance or Confidence
 - But Within The Realm Of Plausibility (???)
 -(any fool can assume that the sky will fall)
 - One That Supports Institutional Resilience
 - One That Will Be Judged By "No Surprises"

STRESS-TESTING REQUIRES

- Truly Severe Values
 - Threats To Survival
 - Firm Not Providing Value If These Are Not "In Sight"?
 - ...(Or Maybe We're Not "Seeing" Very Well?)
 - On Both Extremes
- That Are Somehow Still Plausible
 - By What Standard?
 - History?: at a minimum
 - Theory?: maybe
 - Judgment?: be very wary of setting a maximum
 - Informed By History, Theory, and Judgment

STRESS-TESTING **CAN** / **SHOULD** IGNORE

- Accuracy
 - Around Likely Or Expected Scenarios
- Current Wisdom & Judgment
 - About Variance And Confidence Bands
- The Arbitrage-Free Shibboleth
 - If Someone Couldn't Get Rich Is It Truly Extreme?
 - But Do Preserve Both Extremes
- Risk-Neutral Modeling
 - Risk-Neutral Models Predict Today's Prices
 - Risk-Neutral Distributions Are Make-Believe

Deterministic Interest Scenarios

- Necessary But Maybe Not Sufficient
- Risk That It's Limited By Current Imagination
 - "There Would Be Blood In The Streets"
- Risk That It's Limited By Historical Extremes
 - But If It Already Happened Isn't Worse Plausible?
- How Do We Know How Bad Is Bad Enough?
 - Yet Still Plausible

Back To The Anecdotes - 1978

12% Exceeded The Bounds Of Both History And Imagination



Back To The Anecdotes - 1978 To 1980/81

Imagination and History Were Not Nearly Enough



Deterministic Interest Scenarios

- Necessary But Maybe Not Sufficient
- Risk They Are Limited By Current Imagination
 - "There Would Be Blood In The Streets"
- Risk They Are Limited By Historical Extremes
 - But If It Already Happened Isn't Worse Plausible?
- How Do We Know How Bad Is Bad Enough?
 - Yet Still Plausible
 - OR HOW LONG IS LONG ENOUGH? (Plausibly)

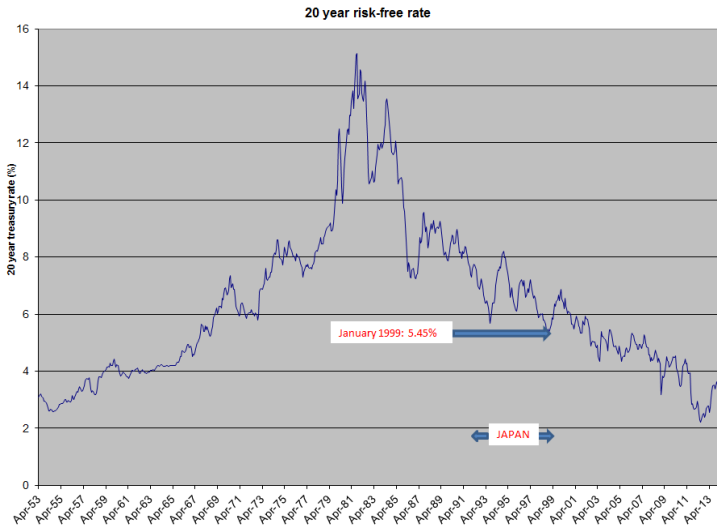
FACTUAL HISTORY

Remember: Double-Digit Rates Only Lasted For 6 Years



Back To The Anecdotes - 1990's

Japan Told Us That $< 2\%$ or 3% Deterministic Was Plausible



Back To The Anecdotes - Flash-Forward

So 2008 & 12 Were Not A "Surprise" For The Deterministic Stress-Test



BUT FOR HOW LONG $< 2\%$ or 3% ?

- Forever?: Not Plausible
(Remember 6 Years Of Double-Digit Rates?)
- 5 - 10 Ten Years?: Maybe Not Severe Enough?
- We Finally Resorted To Random Scenarios
- Definitely A Last Resort
 - We'd Seen Too Much Abuse of Stochastic Models
 - They Only Give Back What You Put In
 - But Hard To Recognize Own Input Coming Back At You
 - No-Arbitrage and Risk-Neutral All The Rage
 - Risked Confusing Even Knowledgeable Audience

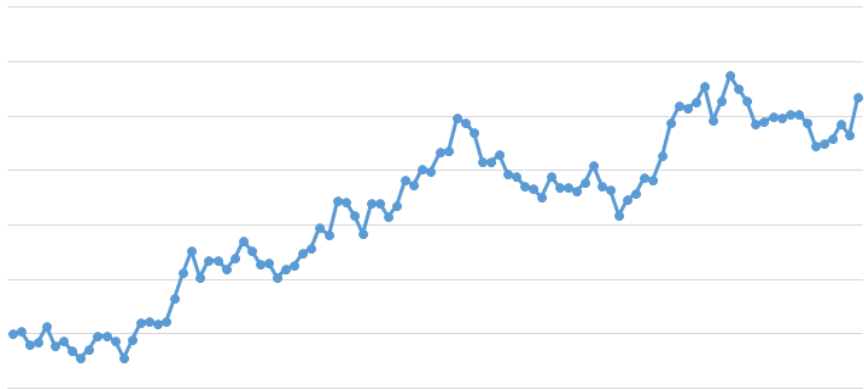
RANDOM INTEREST RATE SCENARIOS

- The Extreme Scenarios Will Be The Stress-Test
 - So Risk-Neutral And Arbitrage-Free Are Irrelevant
- Start With A Model For An Anchor Rate
 - 20 Year Treasury
 - Build A Yield Curve Off That Later
- Choices
 - Pure Dispersion (Random Walk)

The Choices Pictorially

PURE DISPERSION – RANDOM WALK — IMPLAUSIBLE

$$\Delta \ln Rate_t = \text{Gaussian} \Delta$$



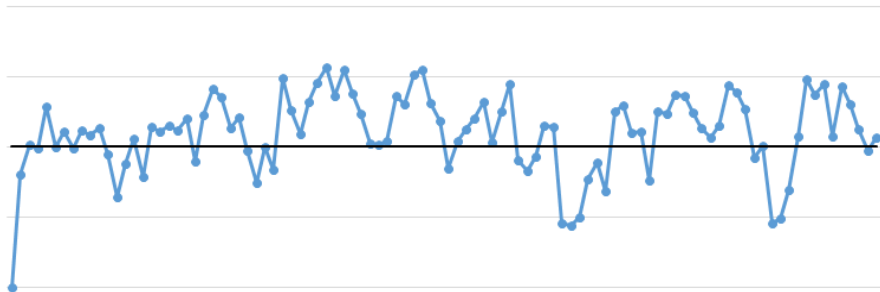
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 - Pure Dispersion (Random Walk) ... Implausible
 - Introduce A Mean Reversion Point (MRP)

The Choices Pictorially

INTRODUCE A MEAN REVERSION POINT (MRP)

$$\Delta \ln Rate_t = F * (MRP - \ln Rate_{t-1}) + (1 - F) * Gaussian\Delta$$



RANDOM INTEREST RATE SCENARIOS

- The Extreme Scenarios Will Be The Stress-Test
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- Choices
 - Pure Dispersion (Random Walk) ... Implausible
 - Introduce A Mean Reversion Point (MRP)
 - Which One (MRP) and How Fast (F)?
 - Any Choices Eliminate Some Historical Extremes –
 - (Either Level Extremes &/Or "How Long?" Extremes)
 - AAA Generator Chose This (And Eliminated Both)

RANDOM INTEREST RATE SCENARIOS

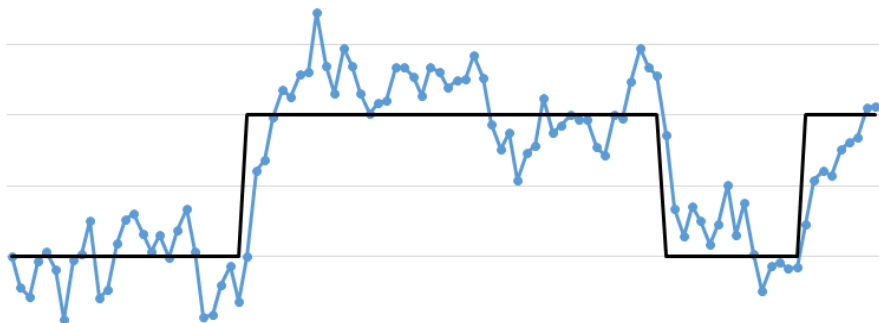
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 - Introduce More Than One MRP (Regimes)
 - Switch Off Among Them (Somehow Randomly)

The Choices Pictorially

INTRODUCE MORE THAN ONE MRP (REGIMES)

$$\Delta \ln Rate_t = F * (MRP_i - \ln Rate_{t-1}) + (1 - F) * Gaussian\Delta$$

$i = 1, 2$ deterministic when regime switch randomly occurs



RANDOM INTEREST RATE SCENARIOS

- More Choices
 - Introduce More Than One MRP (Regimes)
 - Switch Off Among Them (Somehow Randomly)
 - How Many? At What Levels? With What Frequency?
 - Assumptions & Output Both Look Artificial
 - Little Or No Guidance From Interest Rate History
 - ... (How much worse than 15% / 2% is plausible?)

RANDOM INTEREST RATE SCENARIOS

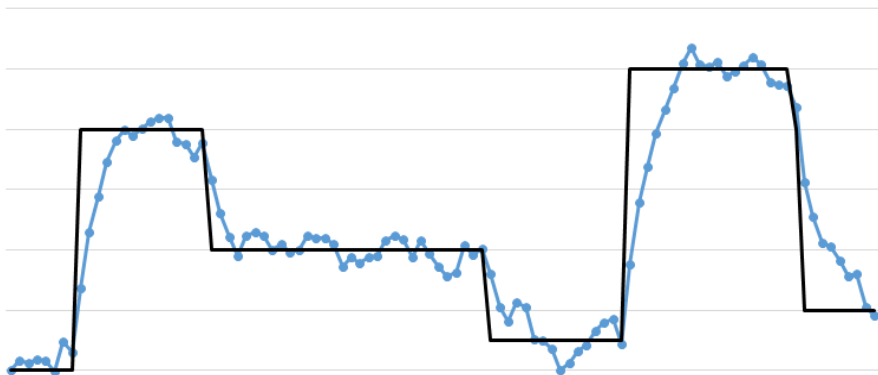
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 - Introduce A Random Field Of MRPs (Regimes)
 - Switch Off Among Them (Somehow Randomly)

The Choices Pictorially

INTRODUCE A RANDOM FIELD OF MRP_s (REGIMES)

$$\Delta \ln Rate_t = F * (MRP_t - \ln Rate_{t-1}) + (1 - F) * Gaussian\Delta$$

MRP_t random when regime switch randomly occurs



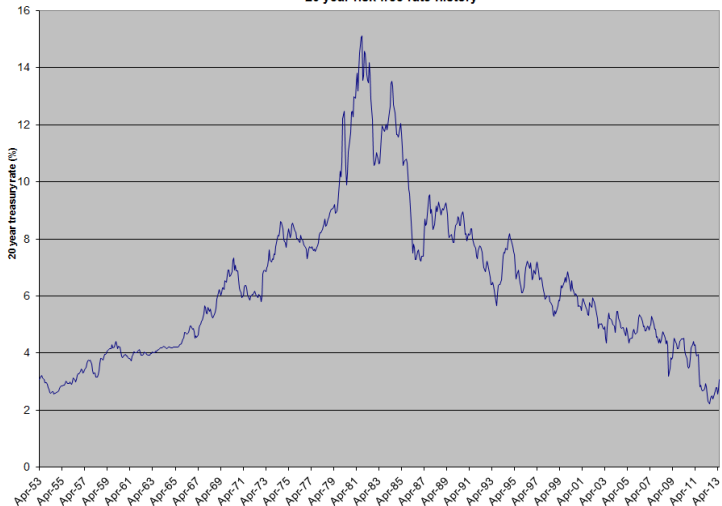
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 - How Many? At What Levels? With What Frequency?
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 - Little Or No Guidance From Interest Rate History
 - ... (How much worse than 15% / 2% is plausible?)
 - Introduce A Random Field Of MRPs (Regimes)
 - Switch Off Among Them (Somehow Randomly)
 - Output Starts To Look Very Natural / Assumptions(??)
 - Historical Extremes Fit Right In
 - We Chose This One – Parameters A Challenge

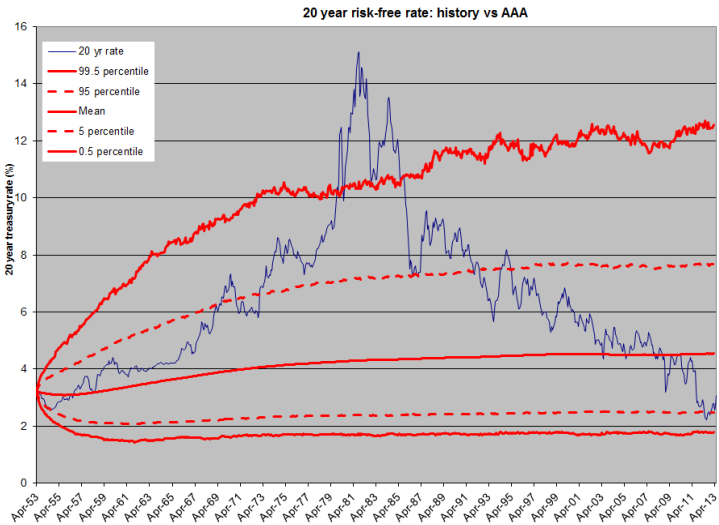
History of 20 Year US Treasury Rate

Plausible By Definition

20 year risk-free rate history



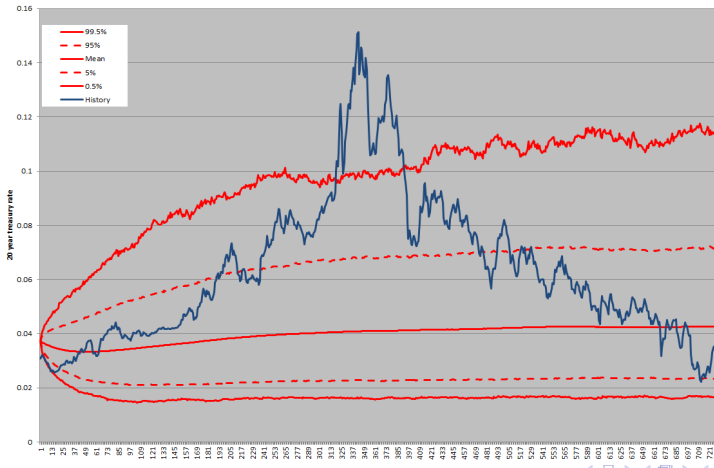
Neither Early 80's Nor Japan Are Remotely Plausible In AAA



M'y %-iles Dec.2013 AAA Generator (NAIC MRP 4.00%)

Update To New MRP Makes It Worse Even With Higher Starting Rate

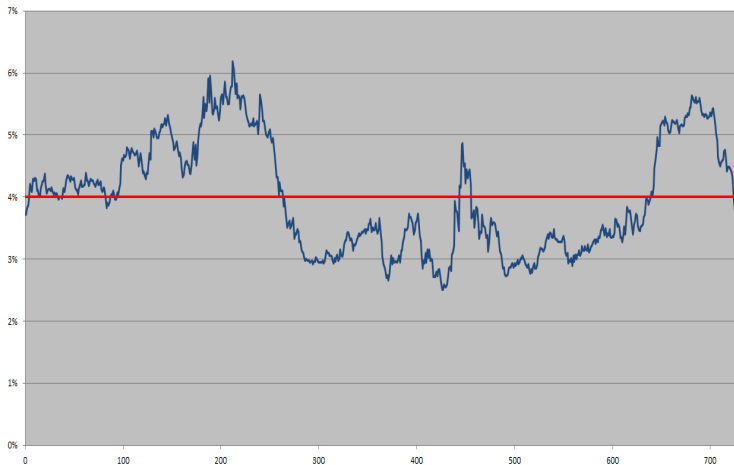
20 yr risk free rate history April 1953 to February 2014 vs AAA starting January 2014



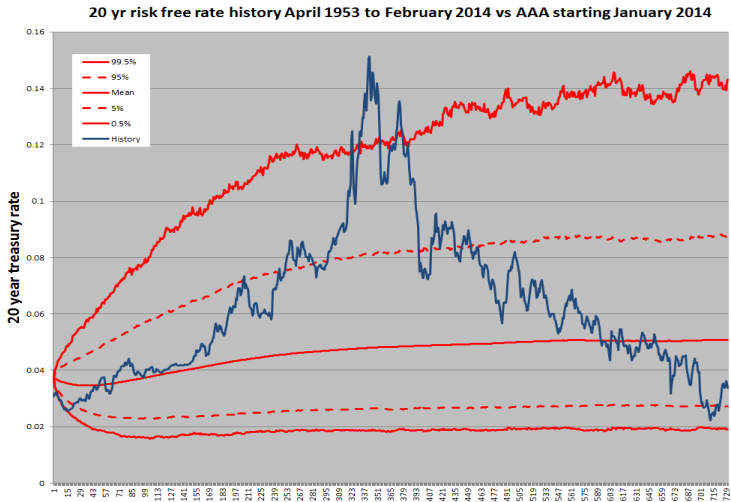
99%-ile Scenario: Dec.2013 AAA (NAIC MRP 4.00%)

No One Scenario Hugs The Bottom – Here's 99% Cumulative < 4% Run

99th Percentile Worst Scenario: No. 9913 - Fixed MRP Eliminates Long Runs < 3% By Its Very Design

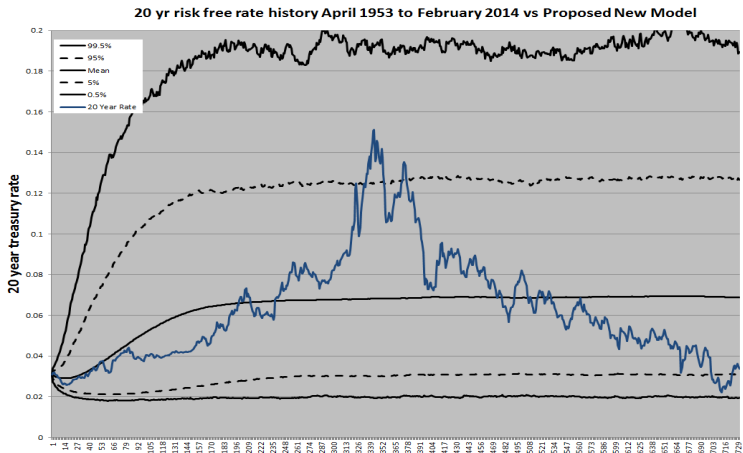


AAA's Recommended MRP Helps A Little, But Loses At The Bottom



M'y %-iles Randomized MRPs / Apr. 1953 Start

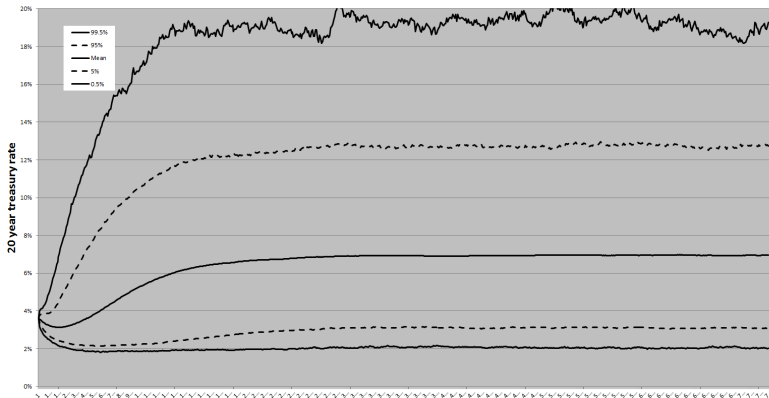
Extreme Enough To Envelop History – But Still Plausible



M'y %-iles Randomized MRPs / Dec. 2013 Start

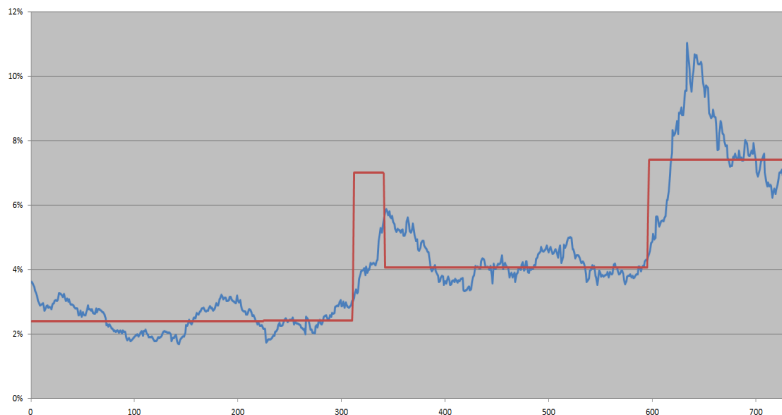
Starts Higher But Still Has Similar Range Of Plausibilities

Proposed New Model Dec. 2013 - 60 Years



And Model Design Has Not Automatically Ruled Out Bottom-Hugging

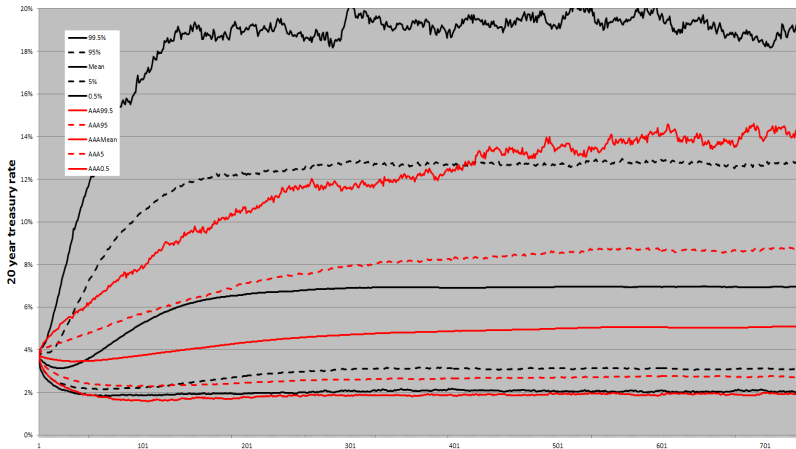
99th Percentile Worst Scenario: No. 866 - Random MRP Opens Up Long Runs < 3%



Randomized MRP vs AAA (with AAA 4.75% MRP)

The High-Rate Risk Is Captured Much Better

Randomized MRP vs AAA 4.75% MRP Dec. 2013 - 60 Years



FROM 1994 TO 2006:

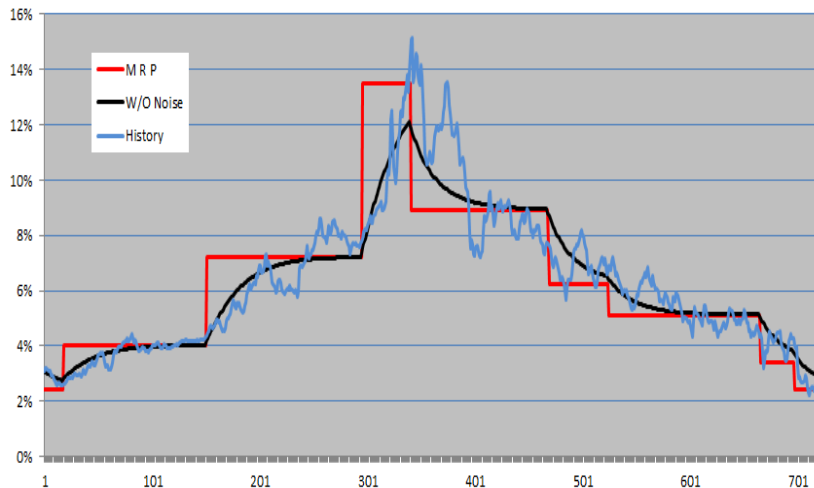
— A LOT OF TRIAL & ERROR

SINCE THEN:

— SOME ATTEMPT AT SCIENCE

Calibration Of The Randomized MRP Model

Historical Regimes – Derived From A Filtering Procedure



WAITING TIME TO REGIME SWITCH

- The MLE Gamma Distribution On Historical
 - Only 8 Data Points
 - $\text{Alpha} = 3.52$; $\text{Beta} = 2.32$
 - $\text{Mean} = \text{Alpha times Beta} = 8.2 \text{ Years}$
 - Alpha And Beta Low Confidence Separately
 - Mean Is Really What Affects Model Output Anyway
 - Interesting That $\text{Mean} = \text{US Political Cycle}$

DISTRIBUTION OF MRP

- Assume Lognormal - Mutually Independent

REVERSION SPEED

- Set Jointly With The Lognormal Parameters
 - To Get Best Fit With Moments Of Historical Rates

Calibration Of The Randomized MRP Model

Rate Levels and Spread Align With History

	61 Year History	Model Mean	Model StdDev
Rate = 20 Year Treasury			
Rate Mean	.0631	.0638	.0131
Rate StdDev	.0266	.0266	.0109
Rate Kurtosis (normal=3)	3.54	2.92	1.24
Rate 6th-osis (normal=15) (6th Ctrl Mom/StdDev^6)	21.7	15.5	19.3

VOLATILITY OF INTEREST RATES

- History Is Not Clearly Lognormal
 - We Fit A Three Parameter DiGeneralized Gamma
 - Using L1 & L2 Distances Of Cumulative Distributions
- Essentially As Good As AAA Generator
 - Which Fits Historical Volatility Very Well Indeed
 - Using Stochastic Volatility & Yield Curve Dynamics

- Many Generations of Actuarial Students
 - At Aetna Life Insurance Company
 - At AnTai Life Insurance Company (in Taiwan)
 - At Aetna International Inc.
- Many Generations of UConn Students
 - Master's In Mathematics, conc. Actuarial Science
 - Master's In Applied Financial Mathematics
- Most Recently
 - Songchen (Darren) Zhang
 - Zepeng (Ben) Xie
 - Xuezhi (Kevin) Zhang
 - Nyan Paing Tin