Risk-Based Valuation

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Statement of the problem

- The CAPM-based capital budgeting theory does not work for capital-constrained firms
  - Idiosyncratic risk may be costly
  - Cash flows in different periods may be correlated
    - (this matters if idiosyncratic risk matters)
  - Benchmarks and comparables should be used when available
  - Using forwards in a CAPM context can be challenging
  - Options and other nonlinear relationships are difficult to include
  - Some CAPM parameters are unknown
    - (e.g. correlation between project and market *return*)
  - Project data normally occur in prices and levels, not returns
- Firms lack an integrated and consistent framework for valuing projects in capital-constrained environments.
  - This presentation uses simulation as a unifying framework to achieve this objective

Outline of this presentation

- Develop consistent framework for CAPM, forwards & options
- General valuation equation for any given risk measure
  - Risk-neutrality and time-neutrality
- Derive pricing formulae with idiosyncratic risk
- Explain the derivation of the cost of risk
- Describe integrated valuation framework

Simplest case: Obtaining the CAPM

- One-year project
- Simulated levels of cash flow (C_1) and market index (M_1)
- Regress C_1 on M_1
  - $C_1 = [\mu_C - \beta_L \mu_M] + \beta_L M_1 + \varepsilon$
  - (note β_L is in levels not returns; $\beta_L = \text{cov}(C_1, M_1)/\text{var}(M_1)$)
- Discount risk-free and market-correlated cash flows assuming residual risk is unpriced
  - $V_0 = [\mu_C - \beta_L \mu_M]/(1+r_f) + \beta_L M_0 + 0$ — Replication pricing
  - $V_0 = [\mu_C - \beta_L\{\mu_M - M_0(1+r_f)\}]/(1+r_f)$ — Risk-neutral pricing
  - $V_0 = \mu_C/(1+r_f) - \beta_L\{\mu_M/(1+r_f) - M_0\}$ — Time-neutral pricing

  } Equiv
- Substitute
  - $\beta_L = \beta V_0/M_0$, $C_1 = V_0(1+r_V)$, $M_1 = M_0(1+r_M)$ — Convert levels to returns
  - $E(r_V) = r_f + \beta(E(r_M) - r_f)$ — CAPM expected return eq.
  - $V_0 = \mu_C/(1+E(r_V))$ — CAPM valuation

Valuing a one-year oil project using forwards

- W=WTI (West Texas Intermediate Crude Oil)
- Simulated levels of cash flow (C_1) and oil prices (W_1)
  - Cash flow depends on revenues and costs, both of which are functions of oil prices
- Regress C_1 on W_1
  - $C_1 = [\mu_C - \beta_L \mu_W] + \beta_L W_1 + \varepsilon$
- Discount risk-free and oil-correlated cash flows assuming residual risk is unpriced and F_W = forward price of oil
  - $V_0 = [\mu_C - \beta_L \mu_W]/(1+r_f) + \beta_L F_W/(1+r_f)$ Replication
  - $V_0 = [\mu_C - \beta_L \{\mu_W - F_W\}]/(1+r_f)$ Risk-neutral
  - Equivalent to time-neutral valuation if $F_W \leftarrow F_W/(1+r_f)$
- Q: What if the relationship between C and W is nonlinear?

Valuing an option in the Black-Scholes framework

- Slight difference: Allow only one rebalancing period, at time zero
- Simulated levels of stock price (S_T) and call option payout (C_T)
  - α = continuous expected growth rate of the stock
- Regress C_T on S_T
  - $C_T = a + bS_T$
    - $b = [E(C_T S_T) – E(C_T)E(S_T)] / [E(S_T{}^2)-E(S_T)^2]$
    - $a = E(C_T) – b\, E(S_T)$
- Discount risk-free and stock-correlated cash flows assuming residual risk is unpriced
  - $C_0 = S_0 \exp[(\alpha-r_f)T]\, N(d_1(\alpha)) – X \exp(-r_f T)N(d_2(\alpha)) – bS_0(\exp[(\alpha-r_f)T]-1)$
    - $d_1(\alpha)$ is the Black-Scholes d_1 with α substituted for r_f
  - Simplifies to Black-Scholes when $\alpha = r_f$

Adapting a general valuation equation

- Every asset satisfies the general valuation equation GVE
  - Expected return = Required return
  - {Exp capital gain} + Exp cash flow = Cash opportunity cost + Risk compensation
  - $\{E[V_{t+1}] - V_t\} + E[C_{t+1}] = r\,V_t + k\,R_{t+1}$ at all times t
- In the CAPM example presented earlier, the risk compensation simplifies to
  - $k\,R_{t+1} = \beta(E(r_M) - r_f)\,V_0 = \beta_L[\mu_M - (1+r_f)M_0]$
- If we move the cost of risk (kR_{t+1}) to the left side of the GVE equation, we obtain risk-neutrality
- The *time-neutral* transformation of cash flows is achieved by discounting all the cash flows and risk measures at the riskless rate and then using an effective riskless rate of 0.
  - Test: Discount cash flows and risk measures at the riskless rate in the GVE and apply a zero discount rate
    - $E[V_{t+1}]/(1+r) - V_t + E[C_{t+1}]/(1+r) = 0 + k\,R_{t+1}/(1+r)$
    - Multiplying by (1+r) and rearranging terms, this produces the original GVE

Pricing idiosyncratic risk

- Normally distributed cash flow C_1 in one year (μ_C, σ_C)
- Apply the GVE:
  - $\{\mu_C - V_0\} + 0 = r_f V_0 + k\sigma_C$
  - $V_0 = [\mu_C - k\sigma_C] / (1 + r_f)$
- Expected return equation
  - $E(r_V) = r_f + k\sigma_C / V_0$
- Same cash flow, but now correlated with the market
- Apply the GVE:
  - $\{\mu_C - V_0\} + 0 = r_f V_0 + \beta_L[\mu_M - (1+r_f)M_0] + k\sigma_\varepsilon$
  - $V_0 = [\mu_C - \beta_L[\mu_M - (1+r_f)M_0] - k\sigma_\varepsilon] / (1 + r_f)$
- The expected return equation
  - $E(r_V) = r_f + \beta[E(r_M) - r_f] + k\sigma_\varepsilon / V_0$

Multiperiod models

- Joint normally distributed cash flows $C_1,\ldots,C_N$ with correlation matrix $\mathbf{R}$, standard deviation vector $\boldsymbol{\sigma}$ and mean vector $\boldsymbol{\mu}$
  - The Cholesky decomposition of **R** is given by **C**, and **I** is the identity matrix
- Make time-neutral conversion for convenience
  - Convert $C_j^* = C_j/(1+r_f)^j$
  - Replace $\mu_j^* = \mu_j/(1+r_f)^j$ and $\sigma_j^* = \sigma_j/(1+r_f)^j$
- Choose risk measure and value
  - Variance of total value (PVAR) $V_0 = \boldsymbol{\mu}^{*\prime}\mathbf{1} - kz\,(\boldsymbol{\sigma}^{*\prime}\mathbf{R}\,\boldsymbol{\sigma}^{*})^{1/2}$
  - Stdev of total value (RPV) $V_0 = \boldsymbol{\mu}^{*\prime}\mathbf{1} - kz\,(\boldsymbol{\sigma}^{*\prime}\mathbf{C}\,\mathbf{1})$
  - Stdev of total value, zero corr (CFAR) $V_0 = \boldsymbol{\mu}^{*\prime}\mathbf{1} - kz\,(\boldsymbol{\sigma}^{*\prime}\mathbf{I}\;\mathbf{1})$

Properties of these models

- Idiosyncratic risk matters
  - hedging adds value
- Correlations between cash flow periods matter
- Ordering of cash flows matters
- Values are non-additive
  - a negative NPV incremental project can add value
- Easy to add market factors (multifactor risk)
- Easy to include option-like payoffs

Determining the private cost of risk (k)

- k is a measure of the adverse impact caused by increased risk
- If an agent accepts a contract or purchases an asset, the incremental risk will generally
  - Add to the risk of the agent's cash flow
  - Increase the risk of declines in future wealth
  - Increase the likelihood of financial distress or bankruptcy
- The value of k is chosen on the margin so the agent is compensated for the cost to his income statement or balance sheet.
- Example
  - Suppose each additional \$100,000 of risk increases the likelihood of financial distress by 5%, and the cost of financial distress is \$250,000. In this case k = expected loss per dollar of risk = (5% of \$250,000)/100,000 = 12.5%.
- Most financial institutions have determined an explicit cost of risk which they use in their valuations of financial assets and contracts.

The consistent framework

- The cash flows of a project along with its traded value drivers can be simulated.
  - Relationships may be linear or nonlinear.
- Time-neutralize cash flows, traded assets and forward prices.
- Regress adjusted cash flows on traded value drivers and compute covariance matrix of residuals.
- Choose the appropriate risk measure.
- Determine the appropriate cost of risk k.
- Value the project using PVAR or RPV.
- Replace NPV criterion:
  - Accept an incremental project if the risk-based valuation of the package exceeds the risk-based valuation of the standalone project.